MATCHED-FIELD PROCESSING:
ACOUSTIC FOCALIZATION WITH DATA TAKEN IN A
SHALLOW WATER AREA OF THE STRAIT OF SICILY

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shallow water area of the Strait of Sicily

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Abstract

Sound is used in underwater applications mainly due to the ocean’s transparency to acoustic waves, whereas it is opaque to electromagnetic radiation. When an array of sensors is used to sample the acoustic field, the measured acoustic pressure appears to be highly spatially dependent both in range and depth. Matched-Field Processing (MFP) is an array processing technique that exploits the complexity of the acoustic field to locate an acoustic source in range, depth and possibly azimuth. This is done by correlating the measured and the model predicted fields for all the source location candidates in a pre-defined grid. If the knowledge about the environmental properties is incomplete, the errors in the MFP output might be such that the source location is missed or (worst) wrongly located. It is possible to overcome this problem by estimating the environmental properties together with the source location. This is a technique close to acoustic tomography that can be viewed as a generalization of MFP and is known in the literature as environmental focalization. In this work incoherent and coherent matched-field processors are studied and compared. The simulations illustrate that the coherent processor outperforms the incoherent processor only if the signal realizations are highly correlated at very low signal-to-noise ratio. During the ADVENT’99 sea trial conducted by SACLANTCEN in May 1999 in the Strait of Sicily, acoustic data comprising three tracks at 2, 5 and 10 km, and on two frequency bands - 200-700 Hz and 800-1600 Hz - were acquired in a shallow water nearly range-independent area (80 m depth). The experimental results indicate that high quality source localization can be obtained for the 5 km track and for the higher frequency band only if the environment is properly focused. In particular, the use of empirical orthonormal functions has shown to be very effective to focus the watercolumn properties, allowing for good localization results.
Resumo

O som é utilizado em aplicações submarinas principalmente devido à transparência do meio submarino a ondas acústicas enquanto que este é opaco às radiações electromagnéticas. Quando uma antena de sensores é utilizada para amostrar o campo acústico, a pressão acústica medida parece ser altamente dependente do espaço, quer em distância, quer em profundidade. Processamento por adaptação do campo (em inglês Matched-Field Processing, MFP) é uma técnica de processamento de antenas que explora a complexidade do campo acústico para localizar fontes acústicas em distância, profundidade, e possivelmente em azimute. Isto é feito correlando o campo medido com os campos preditos para todas as posições da fonte candidatas duma grelha pré-definida. Se o conhecimento sobre as propriedades ambientais for insuficiente, os erros da saída de MFP poderão ser tais que impossibilitem a localização, ou (pior ainda) que esta seja errada. É possível ultrapassar este problema estimando as propriedades do ambiente conjuntamente com a posição da fonte. Esta é uma técnica que está próximo da tomografia acústica que pode ser vista como uma generalização do MFP, e que na literatura é conhecida por focalização ambiental. Neste trabalho processadores de adaptação do campo incoerentes e coerentes são estudados e comparados. As simulações ilustram que o processador coerente tem um desempenho superior àquele alcançado pelo processador incoerente apenas se as diferentes realizações do sinal forem altamente correladas a relações sinal-rudo muito baixas.

Durante a campanha denominada de ADVENT’99 levada a cabo pelo SACLANTCEN em Maio de 1999 no Estreito da Sicília, dados acústicos incluindo três caminhos de propagação, a 2, 5 e 10 km, e duas bandas de frequência - 200-700 Hz e 800-1600 Hz - foram adquiridos numa área de águas pouco profundas praticamente independente da distância (80 m de profundidade). Os resultados experimentais indicam que localização de boa qualidade pode ser obtida quando a fonte está a 5 km e para a banda de frequências mais altas, só se o ambiente for devidamente focalizado. Em particular, o uso de funções empíricas ortonormais mostrou ser muito eficiente na focalização das propriedades propriedades da coluna de água, permitindo bons resultados de localização.
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Chapter 1

Introduction

Underwater acoustics is a scientific discipline that has emerged after the Second World War and is attracting an ever increasing number of multi-disciplinary scientists all over the world. Sound is extensively used in undersea applications simply because the ocean is transparent to the propagation of acoustic waves, whereas it is opaque to the electro-magnetic waves. Underwater acoustics finds application in a variety of problems that include active and passive detection and localization of ships and submarines, high resolution imaging, underwater communications, acoustic tomography, search for buried objects in the seafloor and remote-sensing of natural phenomena, among many others.

A number of these applications were made possible thanks to the recent increase of computation capabilities of mini and personal computers that allow, in nowadays, to accurately solve complex differential systems of equations within reasonable time. If these differential systems of equations properly describe the propagation of sound in the ocean they constitute what is called an acoustic propagation model and thus a powerful tool for predicting sound pressure at any designated location, bringing new perspectives to many applications [8]. In order to be able to solve such complex systems of equations, each model has its own simplification assumptions since they can not cope with the enormous amount of parameters that
characterize the such complex environment (the ocean) where the acoustic signal propagates.

One of the applications where acoustic models have been extensively used in the last thirty years is the passive localization of sound sources. The usage of acoustic models for source localization was first suggested by Hinich [27] and Bucker [6]. The suggested method was simply based on the comparison of the acoustic field received on an array with the acoustic field predicted, by a suitable acoustic model, for the same array but for a given candidate source position. Varying the source position on the acoustic model would produce a series of comparison results forming - what is generally called - an ambiguity function. If the model is sufficiently accurate, relative to the real field, an estimate of the true source location would be given by the maximum of the ambiguity function. That procedure is known as Matched-Field Processing (MFP). Since its introduction in the early 70s, MFP has been intensively studied in many of its aspects (see [2] and references therein).

One of the most important aspects is to determine the impact of incomplete environmental knowledge on the final result. In other words, to quantify what is the necessary accuracy on the environmental data to obtain a correct source location estimate. Another similar problem deals with the accuracy to which the geometry of the receiving array should be known in order to produce a meaningful MFP output. Both missing environmental information and the lack of knowledge of the relative position of the sensors in the array, produce what is known as model mismatch and has been the object of a large number of studies [26, 23, 34, 13, 17]. The bottom line is that the acoustic field is highly dependent on small errors on the sound speed profile (to the fraction of m/s) and on the water depth as well as on the sediment properties in shallow water applications [9, 10]. In terms of sensor position, and as a rule of thumb, sensor depth should be known with an accuracy better than
a fifth of wavelength while the error on sensor horizontal displacement should not exceed half a wavelength [33]. These numbers were drawn from simulated studies and impose severe limitations for real world applications.

Nevertheless, the first single snapshot source localization results with real data were shown in [7, 16]. The first consistent results of source localization along time were shown in [30]. Many other results have been obtained for shallow water data sets [40, 42], in deep water [18], with horizontal arrays with bottom mounted arrays [35], etc. These results were made possible also by the usage of specifically designed processors to counter the model mismatch problem. Conventional methods use the maximum of the correlation between the measured and predicted fields - that is the Bartlett processor. That is known to be the optimum estimator of a single source in white noise; however its noise rejection is poor and with relatively high sidelobes there is little guaranty that the main peak is not overtaken by a sidelobe upon a small error on the environment or on the array geometry. The processors that have been proposed along the years can be classified in two categories: those that are designed to increase sidelobe rejection and those that directly cope with model mismatch in the model itself. The first category are the so-called high-resolution processors like for instance minimum variance based [11], the mode-subspace processor [31], the EMV-PC processor [37] and others. These processors dramatically increase the sidelobe rejection giving very high and well defined peaks. The advantage is that they provide well defined and exact estimates even in high noise conditions. Their disadvantage is that they are generally much more sensitive than the conventional Bartlett processor to model mismatch. The second category of processors take an approach that built the "resistance" to model mismatch into the processor itself. There are at least to well know sub-types of processors in this category:
the uncertain processors (OFUP) [45, 43] and the focalization processor [12, 24, 21, 22].
The OFUP processors takes the environmental model parameters as the mean of an *a priori*
parameter distribution; then a series of conventional processor outputs are generated for a
family of probable models with that distribution. The peak is selected as the OFUP source
location estimate. The focalization processor takes a similar path with, however a significant
difference: the *a priori* distribution is assumed uniform in a given interval; the model is then
adjusted by searching for the maximum output. Both methods are very computer intensive,
depending on the search space dimension the number of acoustic model forward runs can be
of the order of several thousand.

One important issue with mismatch is that it largely depends on frequency: as frequency
increases it becomes more and more difficult to obtain a suitable match in MFP. For a
number of years MFP was confined to narrowband applications, mainly due to the long
computation time required to calculate the acoustic field at several frequencies. In 1988,
Baggeroer *et. al* [1] proposed a broadband processor that was combining the results obtained
at each frequency by direct averaging of the respective ambiguity surfaces. This is known
as the incoherent broadband processor, that was shown to correctly determine the source
location with lower sidelobes than its narrowband counterpart. Since then, other broadband
processors have been proposed in the literature creating however some controversy about the
relative advantages and drawbacks of frequency coherent or frequency incoherent processing
[40]. Frequency incoherent processors are those that combine different frequencies without
taking into account their respective phase terms. Generally the terms that are averaged are
the auto-frequency terms that are shown to contain the most part of the energy; however,
the cross-frequency terms do contain a great deal of information for source localization, but
their combination requires the knowledge of a phase term for in-phase averaging - this is the coherent approach [42].

This work directly addresses the coherent versus incoherent controversy by exploring the properties and extending the efficiency of the recently proposed matched-phase coherent processor [25]. Its performance on terms of noise rejection is theoretically derived and its efficiency in various phase distribution contexts is demonstrated by simulated examples. A simple algorithm is presented to extend its processing capacity to virtually any number of frequency cross terms instead of the three frequency practical limitations mentioned by the authors. Another issue raised by this work concerns the performance of the focalization processor for combating model mismatch in high frequency MFP. This issue is demonstrated when processing real data acquired during the Advent’99 sea trial performed by SACLANT-CEN in the Strait of Sicily in May 1999. During this experiment three vertical arrays where deployed at 2, 5 and 10 km from an acoustic source emitting multi-tones and LFM in two frequency bands of 200-700 and 800-1600 Hz. The environment was nearly range independent and approximately known at all times. Coherent and incoherent MFP, at low and high frequency, at short and long range, with and without focalization has been tested in this data set and are shown in this report.

This report is organized as follows: chapter 2 briefly reviews some topics about acoustic propagation in shallow water like wave equation, normal-modes and ray-solution. In chapter 3 the data model is presented, incoherent and coherent matched-field processors are discussed and compared with synthetic data. Chapter 4 briefly reviews some fundamental aspects of inversion problems and optimization. In chapter 5 a brief description of the experimental setup of the ADVENT’99 sea trial is discussed and the baseline model is presented. In
chapter 6 experimental source localization results, based on the estimated forward models are presented for a source at the three ranges using both frequency bands, are presented and discussed. Finally in chapter 7 taken from this work are presented.
Chapter 2

Acoustic propagation in shallow water

The wave equation in an ideal fluid can be derived from hydrodynamics and the adiabatic relation between pressure and density. Considering that the time scale of oceanographic changes is much longer than the time scale of acoustic propagation, it is assumed that the material properties density $\rho$ and sound speed $c$ are independent of time. The linear approximation of the wave equation involve retaining of only first order terms in the hydrodynamic equations:

$$\rho \nabla \left( \frac{1}{\rho} \nabla p \right) - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0,$$

where $p$ is the acoustic pressure. Note that this is a homogeneous equation, and that $\rho$ and $c^2$ are space dependent. Several numerical methods exist to solve this equation. The major difference between the various techniques is the mathematical manipulation of the wave equation applied before implementation of the solution. In general the task of implementing the solution of the wave equation is very difficult due to the complexity of the ocean-acoustic environment: the sound speed profile is usually non-uniform in depth and range; the sea surface is rough and time dependent; the ocean floor is typically a very complex and rough boundary which may be inclined, and its properties are usually varying over range.
Shallow water is defined as that part of the ocean lying over the Continental Shelf where the water depth is less than 200 m. At frequencies of a few hundred Hz, the shallow water column is of several wavelengths and act as a waveguide whose boundaries are the surface and the bottom. In this type of environment the acoustic field is usually represented by normal modes. The Helmholtz equation is the wave equation in the frequency domain, and can be written in cylindrical coordinates under the assumption of cylindrical symmetry as:

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) + \rho(z) \frac{\partial}{\partial z} \left( \frac{1}{\rho(z)} \frac{\partial p}{\partial z} \right) + \frac{\omega^2}{c^2(z)} p = -\frac{\delta(r)\delta(z-z_s)}{2\pi r}.
\]

(2.2)

Using the technique of separation of variables the solution being searched has the form \( p(r, z) = \Phi(r)\Psi(z) \). Replacing this in (2.2) and after some manipulations,

\[
\rho(z) \frac{d}{dz} \left[ \frac{1}{\rho(z)} \frac{d\Psi(z)}{dz} \right] + \left[ \frac{\omega^2}{c^2(z)} - k^2_{rm} \right] \Psi(z) = 0,
\]

(2.3)

with \( k^2_{rm} \) denoting the separation constant

\[
k^2_{rm} = \frac{1}{\Phi(r)} \left[ \frac{1}{r} \frac{d}{dr} \left( r \frac{d\Psi}{dr} \right) \right],
\]

(2.4)

and \( \Psi_m \) denotes a particular function \( \Psi \) obtained with \( k_{rm} \), and denote the modes which build a complete set. The modal equation (2.3) is to be solved with the appropriate boundary conditions, and since the \( \Psi_m \) form a complete set of functions, the pressure can be represented as

\[
p(r, z) = \sum_{m=1}^{\infty} \Phi_m(r)\Psi(z)_m
\]

(2.5)

Thus the solution yields

\[
p(r, z) \approx \frac{i}{4\rho(z_s)\sqrt{8\pi r}} e^{-i\pi/4} \sum_{m=1}^{\infty} \Psi_m(z_s)\Psi_m(z) e^{ik_{rm}r} \sqrt{k_{rm}}.
\]

(2.6)

In reality the wavenumber spectrum is composed by a continuous and a discrete part, corresponding to evanescent and radiating spectrum respectively. The solution in (2.6) is obtained
under the assumption that the spectrum is composed only by the discrete part. Hence the solution is valid only at ranges greater or equal than several water depths away from the source.

An alternative approximation to the wave equation is the so called "high frequency approximation" that consists in representing the acoustic field by the ray solution. The ray solution of the wave equation is a high frequency approximation, that is useful particularly for deep water problems, where generally only a few rays are significant. Ray tracing is satisfactory if the wave length is much less than the length scales in the problem. In shallow water many significant rays arrive to the receiver, whereas the modes are only a few, which implies that mode models are preferable to ray tracing models. For ray tracing Snell’s law provides a simple formula for calculating the ray declination angle when the channel is modeled as a stratified medium based on the knowledge of the soundspeed at the interface between to layers.
Chapter 3

Matched-Field algorithms

3.1 The Data Model

A signal transmitted by a source, exciting a horizontally stratified, parallel waveguide, is received by a vertical sensor array and observed during time $T$. The signals received at each sensor are included in a vector $x(t)$, for $0 < t \leq T$. The channel is assumed as being linear, causal, and time-invariant; $s(t)$ is a scalar representing the signal at the source, and $h(t) = h(t, \vartheta)$ is a vector with elements $h_n(t)$ representing the channel’s impulse response between the source and sensor $n$, i.e., the solution of the Helmholtz equation, also called Green’s function; $u(t)$ denotes additive stationary noise. Thus $x(t)$ is defined by the convolution equation:

$$x(t) = \int_{0}^{\infty} h(t - \tau) s(\tau) d\tau + u(t). \quad (3.1)$$

The Fourier transform of the vector $h(t)$ is the transfer function vector $H(\omega) = H(\omega, \vartheta)$ which is known except for the parameter vector $\vartheta$:

$$H(\omega, \vartheta) = [G(\omega, \vartheta, r_1) \ldots G(\omega, \vartheta, r_N)]^T, \quad (3.2)$$

where $G$ is the solution of the Helmholtz equation, and $r_i$ is a vector that designates the $i^{th}$ sensors location. To work with finite data portions in the Finite Fourier Transform domain allows for assumptions of asymptotic distributions for the transformed data for increasing
observation times. It is assumed a band limitation $|\omega| < \Omega$ for the signal and noise. The output of the sensors is sampled at a rate $\omega_s > 2\Omega$, i.e. a sampling period $\Delta < \pi/\Omega$. The Fourier Transform is given by

$$X(\omega) = \Delta \sum_{i=1}^{N} w_T(\Delta i) x(\Delta i) e^{-j\omega \Delta i}. \quad (3.3)$$

The window $w_T(t)$ is zero outside the interval $[0, T]$, and normalized, $\int_{-\infty}^{\infty} w_T(t)^2 dt = T$. Thus, (3.1) is written in the frequency domain as

$$X(\omega) = H(\omega) S(\omega) + U(\omega), \quad (3.4)$$

where $S(\omega)$ is the scalar deterministic source waveform, and $U(\omega)$ is $N(0, \sigma_U^2 I)$ distributed. Note that although no statistical assumption has been made on $u(t)$ (only stationarity has been assumed), it is possible to assume a distribution for $U(\omega)$ - this is a consequence of the central limit theorem.

In order to model ocean inhomogeneities, a stochastic complex factor $p(\omega)$ is introduced in the signal term leading to

$$X(\omega) = H(\omega) S(\omega) p(\omega) + U(\omega), \quad (3.5)$$

At this point, no considerations are made on the statistical properties of $p(\omega)$, and this shall be discussed later on in more detail. It is defined as

$$p(\omega) = e^{j\phi_H(\omega)}, \quad (3.6)$$

where $\phi_H(\omega)$ is a random variable whose distribution establishes the distribution of $p(\omega)$.

Under the assumption that signal and noise are uncorrelated, noise is uncorrelated from sensor to sensor, and $X(\omega)$ is normally distributed and zero mean, its covariance matrix is

$$C_{XX}(\omega) = \text{E}[X(\omega)X^H(\omega)] = |S(\omega)|^2 H(\omega) H(\omega)^H \text{E}|p(\omega)|^2 + \sigma_U^2 I. \quad (3.7)$$
For some methods it may be important for frequencies \(0 < \omega_1 < ... < \omega_N < \Omega\), to have \(X(\omega_1), ..., X(\omega_N)\) asymptotically independent. It can be shown that independent random variables are uncorrelated. For example for methods using cross-frequency correlation it is required to characterize the moment of second order involving signals at different frequencies.

The noise can be considered as being independent from frequency to frequency, i.e., uncorrelated. The same does not happen to the perturbation \(p(\omega)\), since it represents changes that occur in the propagation channel, and has a certain impact in the whole spectrum of interest.

It will be assumed that this factor is slowly changing over frequency. Hence the assumption of high correlation in a limited band may be valid. The cross-frequency covariance matrix is defined by

\[
C_{XX}(\omega_i, \omega_j) = E[X(\omega_i)X^H(\omega_j)] = S(\omega_i)S^H(\omega_i)H(\omega_j)H^H(\omega_j)E[p(\omega_i)p(\omega_j)], i \neq j
\]  

(3.8)

where \(E[p(\omega_i)p(\omega_j)]\) is the covariance between the perturbations at frequencies \(\omega_i\) and \(\omega_j\).

Note that cross-frequency covariance resulted in a matrix that contains only a signal component: due to the assumption of uncorrelated noise between frequencies, the noise component has vanished.

3.1.1 Data model perturbation: the statistical approach

Section 3.1 presented the data model that will be used throughout this work. This is the linear data model, as it usually is assumed. There is one aspect that may be of some relevance: in the signal term appeared a factor that was called perturbation, and somehow intends to represent the random features of the channel. Thus, considerations relative to the statistical properties of this factor should be drawn. Note that in the model, the perturbation
$p(\omega)$ is frequency dependent. Let $p$ be a complex scalar defined as

$$p = p_X + jp_Y. \quad (3.9)$$

The question that arises is which is the statistical distribution of the phase of such perturbation. It is commonly accepted to assume the phase of factor $p$ to be uniformly distributed. Considering the interval between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$, for an uniform distribution the mean is zero, and the variance is $\frac{\pi^2}{12} \approx 0.82$. Having this in mind it might be illustrative to observe the real data acquired during the ADVENT’99 Sea Trial to get some insight on this distribution. Taking the acoustic pressure received by a given sensor during a limited time, the phase at a certain frequency can be observed.

Fig. 3.1 shows how much the phase increases from one instant to the other on sinusoidal signals at 800 Hz transmitted during 10 seconds (circles), where measurements are done for each 0.5 s. These signals were divided into segments of 0.5 seconds, where the first and the last segments were discarded, hence giving 18 segments. Each of them was Fourier transformed in order to extract the phase at the frequency of interest. Since the variability is low, it can be noticed that the phase is approximately linear. It is interesting to obtain a statistical analysis of this variation in order to get some insight on its distribution. Thus all the available phase observations were obtained as explained above and consecutive phase terms were subtracted. In order to obtain the phase what has been called perturbation these phase differences were subtracted by the slope of the squares line in fig. 3.1. Fig. 3.2 shows the histogram obtained at 800 Hz. The deviation between the measured histogram (continuous line) and a Gaussian distribution with mean -0.136 and standard deviation of 0.188 radians (dashed line) is minimal. This data set shows some disagreement with what is commonly assumed: the distribution for the phase perturbation is normal, whereas the
assumption commonly taken is a uniformly distributed phase perturbation. Note that to assume an uniform phase for the perturbation is equivalent to assume real and imaginary parts to be gaussian.

Thus the problem that appears now concerns the distribution of the real and imaginary parts of the perturbation when the phase has normal distribution. The complex phase can be written as

\[ p = \cos X + j \sin X, \]  

where in this case \( X \) is a random variable (RV) with normal distribution. Let \( Y \) be a RV defined by

\[ Y = \cos X \]  

Figure 3.1: Examples of phase variations in time intervals of 0.5 s for sinusoidal transmissions of 800 Hz acquired during the ADVENT’99 sea trial.
3.1. THE DATA MODEL

Figure 3.2: Continuous line: histogram of the phase perturbation in the acoustic data acquired during the ADVENT’99 experiment. Dashed line: theoretic line where the mean is set to -0.136 and the standard deviation to 0.188.

The distribution function of $Y$ is given by

$$F_Y(y) = \Pr\{Y \leq y\} = \Pr\{\cos X \leq y\} = \Pr\{X \leq \cos^{-1} y\}$$

$$= F_X(\cos^{-1} y)$$

$$= \int_{-\pi}^{\cos^{-1} y} f_X(x) dx,$$  \hspace{1cm} (3.12)

those results were obtained using the definition of the distribution function. Note that $X \in [-\pi, 0]$, since it is required to have a monotonic growing function in the interval of interest. Replacing $y_1 = \cos x$ for $x$ it comes out that $dx = (1-y^2)^{-\frac{1}{2}} dy_1$, and the integration is made between $-1$ and $y$:

$$F_Y(y) = \int_{-1}^{y} \frac{f_X(\cos^{-1} y_1)}{\sqrt{1 - y_1^2}} dy_1,$$  \hspace{1cm} (3.13)

where $y \in [-1, 1]$. Derivating with respect to $y$, the final probability density function is obtained:

$$f_Y(y) = \frac{f_X(\cos^{-1} y)}{\sqrt{1 - y^2}}.$$  \hspace{1cm} (3.14)
If $X$ is zero mean ($m = 0$), and has variance $\sigma^2$ then its density probability function is

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{x^2}{2\sigma^2}\right).$$  \hspace{1cm} (3.15)

The distribution obtained in (3.14) is not found in the literature but is related to the normal distribution. Furthermore, the density probability function for the imaginary part is a symmetric function very similar to the Gaussian function:

$$f_Y(y) = \frac{f_X(\sin^{-1} y)}{\sqrt{1 - y^2}}.$$ \hspace{1cm} (3.16)

Fig. 3.3 shows examples of the new distribution obtained (continuous line) in comparison with Gaussian functions (dashed line). Fig. 3.3a) shows (3.14) for standard deviation $\sigma$ equal 0.4 and mean $m$ equal 0. Using the minimum square error criteria it was found that the Gaussian function that best fits the real part density probability function is the one with mean equal 2 and variance equal 0.24. Note that the domain of interest is restricted to the domain of the function $\sin^{-1} y$. In fig. 3.3b) (3.16) is compared to a Gaussian function having $\sigma$ equal to 0.41. The fitting is quite significant.

![Graphs showing distributions](image)

Figure 3.3: Continuous lines: density probability functions of the (a) real and (b) imaginary parts of complex numbers with uniform phase. Dashed lines: Gaussian functions that best fit to the continuous lines.

In the literature it is not clear whether it is the distribution of the rectangular components
that is independent or whether it is the distribution of the phase. This result suggests that for practical use, the functions can be considered equivalent, if it is the distribution of the rectangular distribution that is assumed independently.

3.2 Conventional matched-field processing

3.2.1 The incoherent conventional processor

The incoherent conventional processor, also called linear processor, has been used in almost every work. The idea is to measure the correlation between the measured field and the replica field making a linear combination of the acoustic pressure received in each sensor through the elements of the replica vector:

\[
Y(\omega, \vartheta) = \sum_{n=1}^{N} w_n^*(\omega, \vartheta) X_n(\omega) = w^H(\omega, \vartheta) X(\omega),
\]

where \(w\) is a weighting vector, and \(X_n\) is the pressure at sensor \(n\). Basically (3.17) is a spatially coherent weighted sum of the pressure measured across the sensors. The energy detector is based on the mean power at the processor output, thus

\[
P(\omega) = E|Y(\omega, \vartheta)|^2
= w^H(\omega, \vartheta) E[X(\omega) X^H(\omega)] w(\omega, \vartheta)
= w^H(\omega, \vartheta) C_{XX}(\omega) w(\omega, \vartheta).
\]

The goal now is to determine what is the vector \(w\) that maximizes \(P(\omega)\). To simplify the notation the dependencies of \(w\) and \(H\) are supressed. Thus maximization proceeds as follows

\[
\max_{w} \{w^H C_{XX}w\} = \max_{w} \{\sigma^2_S w^H H^H H w + \sigma^2_N w^H w\}
= \max_{w} \{\sigma^2_S |w^H H|^2 + \sigma^2_N\}.
\]
To obtain a non-trivial solution, the norm of \( w \) is constrained to 1 when carrying out the above maximization. In (3.19) the second term is a non-negative constant, thus only the first term has to be maximized. The estimator for \( w \) obtained by maximizing (3.18) is

\[
w = \frac{H}{\sqrt{H^H H}}.
\]

(3.20)

Taking into account the data model and (3.17) it can be noticed that \( w \) is a matched-filter, since the filter is exactly the normalized channel’s response. However the only thing that is known is the structure of the filter, remaining the parameter \( \vartheta \) to be estimated. Finally, and once the estimator for \( w \) has been obtained, it can be placed back into (3.18):

\[
P(\omega, \vartheta) = \frac{H^H(\omega, \vartheta)C_{XX}(\omega)H(\omega, \vartheta)}{H^H(\omega, \vartheta)H(\omega, \vartheta)}.
\]

(3.21)

This processor has into account only one frequency. In the last years it became common to use the broadband processor [1]:

\[
P(\vartheta) = \frac{1}{K} \sum_{k=1}^{K} \frac{H^H(\omega_k, \vartheta)C_{XX}(\omega_k)H(\omega_k, \vartheta)}{H^H(\omega_k, \vartheta)H(\omega_k, \vartheta)}.
\]

(3.22)

By processing several frequencies together it is expected to obtain a reduction of the variance of the estimates by reducing the variance of noise, and the estimates obtained are valid for a band of frequencies - at least in the neighborhood of the frequencies being used [47]. The sum in (3.22) is carried without having into account the phases of the signals involved in each correlation term - it is an incoherent processor. In practice only sample covariance matrices are available:

\[
\hat{C}_{XX} = \sum_{l=1}^{L} X_l(\omega)X_l^H(\omega),
\]

(3.23)

therefore an estimate \( \hat{P}(\vartheta) \) of (3.22) is obtained.

Later it will be useful to have the matched-field response obtained by the incoherent
processor for true parameter \( \vartheta \). This is obtained using the data model defined in (3.5), (3.7) can be replaced in (3.18), and \( w \) given by (3.20):

\[
P = w^H C_{X X} w
\]

\[
= w^H (|S|^2 H H^H E|p|^2 + \sigma_U^2 I) w
\]

\[
= |S|^2 E|p|^2 + \sigma_U^2 w^H w
\]

\[
= |S|^2 E|p|^2 + \sigma_U^2. \quad (3.24)
\]

3.2.2 The coherent conventional processor

The previous section presented a processor that takes into account the output of the array’s sensors in a coherent fashion. However, each term in the summation of (3.22) is a real scalar, which means that the sum is incoherent. Taking again the data model in (3.5) and the data vector

\[
X_{coh} = \frac{1}{L} [X(\omega_1)^T \ldots X(\omega_L)^T]^T, \quad (3.25)
\]

a processor that combines coherently different frequencies can be derived. The \( H(\omega_i) \) can be joined into a vector like:

\[
H_{coh} = [H^T(\omega_1), \ldots, H^T(\omega_L)]^T.
\]

For convenience the vectors \( H(\omega_i) \) have norm equal 1. Now the steps to be followed are identical to those taken to derive the incoherent estimator (3.17)-(3.20), and the result is

\[
w_{coh} = H_{coh}
Note that the covariance matrix in (3.18) is now a block matrix containing the matrices $C_{XX}(\omega_m, \omega_n)$ and is denoted by $C_{coh}^{XX}$. Replacing again in (3.18) gives:

$$P(\vartheta) = H_{coh}^H C_{coh}^{XX} H_{coh}$$

$$= \frac{1}{L^2} \sum_{n=1}^{L} \sum_{m=1}^{L} H(\omega_n, \vartheta) C_{XX}(\omega_n, \omega_m) H(\omega_m, \vartheta)$$

(3.26)

In general, each term is a complex number, except for $m = n$. But since frequency $\omega_m$ is combined with frequency $\omega_n$ and vice-versa (one correlation term is complex conjugated of the other), the result of the sum of these two terms is twice the real part of one of the terms. For example in a range-depth ambiguity surface, it is guaranteed that only for the true location each term of the sum is a real number, since there is a perfect match between the measured field and replica field. For all the other locations on the ambiguity surface each term of the sum will be complex. Hence part of the energy is discarded, reducing the sidelobes.

In the last part of the previous section the peak value for the true parameter and uncorrelated noise is given by (3.24). The same is done for one term corresponding to the cross-covariance matrix $C_{XX} = C_{XX}(\omega_i, \omega_j)$ of (3.26):

$$P = w_i^H C_{XX} w_j$$

(3.27)

where $w_i$ is the replica for frequency $\omega_i$, and $C_{XX}$ is the matrix obtained in (3.8). Replacing $C_{XX}$ by its form (3.8) as in (3.24), the following result for the power is obtained:

$$P = S_i S_j^* \text{E}[p_i p_j^*]$$

(3.28)

where the indexes $i$ and $j$ are relative to the frequencies $\omega_i$ and $\omega_i$ respectively. This result indicates that if $p_i$ e $p_j$ are uncorrelated, then the peak will be low. In general the peak for the coherent processor will be always equal or lower than that obtained by the incoherent
processor due to the contribution of the cross-frequency terms and applying the Schwarz inequality:

$$E|p_i|^2 \geq E[p_ip_j^*].$$

(3.29)

The expectance in (3.28) is independent of the parameters $\vartheta$ being searched. Hence, all points in the surface suffer an attenuation by the same factor, i.e., an attenuation of the main lobe implies an attenuation of the sidelobes. As it was shown in section 3.2.1, the noise power has a constant contribution over the whole surface, hence increasing the level of the ambiguity surface by a constant. Further, the number of terms of the coherent processor is quadratic relative to the number of frequencies, which means that the number of ambiguity surfaces to average is much larger, which may be an additional factor contributing for sidelobe reduction.

3.2.3 The matched-phase coherent processor

In section 3.2.2 was developed a coherent processor accounting for frequency coherence. However, this processor does not account for the phase relationships between frequencies. In section 3.1 was introduced the data model with a perturbation term $p(\omega)$ defined in (3.6) as a complex exponent. The intention is to model an uncertainty in the phase of $H(\omega)$ that can not be predicted by the model. The idea now is to derive a processor that compensates these phase uncertainties and simultaneously accounts for the phase relationships between frequencies. Thus, instead of using simply the replica produced by a propagation model like that of (3.25), a complex exponential factor is introduced

$$H_{MP} = [H_T(\omega_1)e^{j\phi_H(\omega_1)}, \ldots, H_T(\omega_M)e^{j\phi_H(\omega_L)}]^T,$$

in order to compensate the phase obtained the mathematical expectancy in 3.8.
3.2. CONVENTIONAL MATCHED-FIELD PROCESSING

Following the same steps as in section 3.2.2, and using (3.30) the matched-phase processor is obtained:

\[
P(\bar{\theta}) = H_{MP}^H C_{XX}^{coh} H_{MP}
\]

\[
= \frac{1}{L^2} \sum_{n=1}^{L} \sum_{m=1}^{L} H(\omega_n, \bar{\theta}) C_{XX}(\omega_n, \omega_m) H(\omega_m, \bar{\theta}) e^{j[\phi_H(\omega_m) - \phi_H(\omega_n)]}
\]  

(3.30)

This processor works in the same way as the coherent processor shown in section 3.2.2, however, with the difference that a cross-frequency dependent phase factor \(\phi_H(\omega_m) - \phi_H(\omega_n)\) is introduced. Instead of losing the energy in the imaginary part of each correlation term in 3.30, this is transferred to the real part. To allow this, it is required that the phase of the exponent factor is the symmetric phase of the correlation factor. Then all the terms turn into real numbers, and the sum is carried out in phase. This gives the possibility of improving the peak-to-sidelobe ratio by adjusting the phase factors. Complex numbers can be represented in a complex plane as vectors, and it is known that the sum of two vectors has maximum norm when they have the same direction. In [25] it is suggested to search for the relative phases as free parameters between 0 and \(2\pi\). After the reasoning above this is not necessary, as it is possible to dramatically restrict the search space. In the next section a much more efficient implementation of this processor will be shown.

3.2.4 Efficient implementation of the matched-phase coherent processor

The matched-phase processor treated in section 3.2.3 was presented in [25]. It was mentioned that the search for the relative phases would be burdensome if the number of frequencies were greater than three. Also Michalopoulou [42] discusses the possibility of looking for unknown phases as free parameters, but abandons this strategy for the same reason. The goal is to look for the phase relationships between different frequencies, i.e. to look for
the phases $\phi_H(\omega_i)$ in (3.30). The number of terms in (3.30) is quadratic with the number of frequencies, and for each cross-frequency term a phase term is to be searched as a free parameter. Thus, if $L$ is the number of frequencies, the number of free phase terms to be searched is $L(L - 1)$, increasing rapidly the dimensionality of the search procedure, a side effect that is not desirable. For illustration, for $L = 3$ the number of relative phases to be searched is 6, but if $L = 7$ then the number of relative phases is already 42. The understanding of how the processor works gives the possibility of an efficient implementation in order to dramatically reduce the computational cost. Here, a new approach for calculating the necessary phase terms is given. First of all one can make use of the following:

\[
\begin{align*}
    p_{12} &= e^{-i[\phi_1 - \phi_2]}w_1^H C_{12}w_2, \\
    p_{13} &= e^{-i[\phi_1 - \phi_3]}w_1^H C_{13}w_3, \\
    p_{21} &= e^{-i[\phi_2 - \phi_1]}w_2^H C_{21}w_1, \\
    p_{23} &= e^{-i[\phi_2 - \phi_3]}w_2^H C_{23}w_3, \\
    p_{31} &= e^{-i[\phi_3 - \phi_1]}w_3^H C_{31}w_1, \\
    p_{32} &= e^{-i[\phi_3 - \phi_2]}w_3^H C_{32}w_2,
\end{align*}
\]

where the notation for the cross-frequencies covariance matrix has been changed for convenience to $C_{ij}$, indicating that frequencies $i$ and $j$ are involved. Thus,

\[
\begin{align*}
    p_{12} &= p_{21}^*, \\
    p_{13} &= p_{31}^*, \\
    p_{23} &= p_{32}^*.
\end{align*}
\]

The sum of a complex number with its conjugate gives twice the real part of the number. The terms involved in the sum to be carried out are the $p_{ij}$ and their complex conjugated.
Thus, the sum of all terms is equivalent to the sum of twice the real part of terms where $i < j$:

\[
P_{MP} = p_{12} + p_{21}^* + p_{13} + p_{31}^* + p_{23} + p_{32}^*
\]

\[
= 2\text{Re}\{p_{12}\} + 2\text{Re}\{p_{13}\} + 2\text{Re}\{p_{23}\}.
\]

(3.31)

At first sight this seems to be a small gain. However, the number of terms in this sum increases quadratically with the number of frequencies, hence for a high number of frequencies this is indeed a significant gain.

But the most important part concerns the search of the relative phases. Assuming a case where the search is being done in range and depth, it is known that the phases that maximize the power are symmetric of the phase of the $w_i^H C_{ij} w_j$, i.e.,

\[
\phi_i - \phi_j = -\angle w_i^H C_{ij} w_j
\]

(3.32)

where $w_i$ and $w_j$ are the replicas for the correct location parameters. However, the correct location parameters are unknown. The first step is to compute ambiguity surfaces,

\[
s_{ij}^{mn} = w_i^H C_{ij} w_j
\]

(3.33)

scanning $w_i$ and $w_j$ against range and depth. The superscripts $m$ and $n$ stand respectively for depth and range. After this step, a set of complex numbers is available for each location (range and depth). It is known that the phase terms that maximize (3.31) are such that the $s_{ij}$ for the correct location are all real numbers. For this specific location, to compute $P_{mn}$ ($m$ and $n$ are indexes for range and depth respectively) as defined in (3.31) with the phase terms that give the maximum is equivalent to

\[
P_{mn} = 2|w_1^H C_{12} w_3| + 2|w_1^H C_{13} w_2| + 2|w_2^H C_{23} w_1|.
\]

(3.34)
In other words either (3.31) or (3.34) yield the maximum for the same location and for a certain set of phase terms. Thus, (3.34) can be scanned versus range and depth in order to obtain its maximum, which leads directly to the phase terms being searched by applying (3.32) for the \( P_{mn} \) corresponding to the maximum. Let \( S_{ij} \) be a matrix corresponding to the frequencies \( i \) and \( j \) where the element in column \( m \) and \( n \) is \( s_{ij}^{mn} \) defined in (3.33), and \( \Phi \) a matrix with the phases selected by maximization of (3.34). The final step is to linearly combine the ambiguity surfaces \( S_{ij} \), and take the maximum:

\[
P_{MP} = \max \left\{ \frac{1}{L(L-1)} \sum_{i<j}^{L} 2 \text{Re}\{S_{ij} e^{\Phi_{ij}}\} \right\}.
\] (3.35)

The algorithm requires the following steps are taken:

1. Compute the two-frequency ambiguity surfaces for all frequency pairs.

2. Sum the absolute values of the ambiguity surfaces obtained in step 1.

3. Obtain phases corresponding to the maximum obtained in the step 2.

4. Combine ambiguity surfaces through the symmetric of the phases obtained in step 3 (only the real parts multiplied by two).

To look exhaustively for the phase terms would require to take \( K \) values in the interval from 0 to \( 2\pi \) for each frequency. In a case where range and depth were discretized into respectively \( M \) and \( N \) samples and for \( L \) frequencies, this would result in a search space with size \( O = K^L \times M \times N \). With the new approach the phases are determined through direct inspection of the correlation terms - in the total only two ambiguity surfaces have to be computed, independently of the number of frequencies.
3.3 Incoherent vs. coherent using synthetic data

Sections 3.2.1 and 3.2.2 respectively presented incoherent and coherent conventional matched-field processors. Section 3.2.3 gives a further development of the coherent processor that accounts for phase relationship of the acoustic pressure at different frequencies, improving its ability for sidelobe reduction. While the incoherent processor suffers of high sidelobes if the noise level is high, the coherent processor has the ability to completely reject the noise, but on the other hand, it requires the signals at different frequencies to be highly correlated.

Incoherent and coherent processors are tested with synthetic data for different signal-to-noise ratios, different number of frequencies, and different distributions of the perturbation phase $p(\omega)$. The goal of these tests is to illustrate the conclusion drawn in the previous paragraph, where the attention goes mainly to the perturbation $p(\omega)$, since this determines whether the different signal realizations are highly correlated or not. The incoherent processor is compared with the coherent matched-phase processor, where the latter considers only the cross-frequency terms as done in [25] ($m \neq n$). This option makes sense, since the sum of the single-frequency terms by itself is the incoherent processor.

Synthetic data was generated using the SACLANTCEN normal mode propagation model C-SNAP [15], and the scenario chosen is similar to that in the Strait of Sicily where the ADVENT’99 sea trial took place. The simulations consider an array of 32 sensors, and the frequencies range from 300 to 600 Hz. The number of signal realizations generated for estimating the covariance matrices was 32. To measure the performance of each of these processors the criteria chosen is the probability of correct source localization versus signal-to-noise ratio (SNR). SNR varies from values where the probability of localization is zero or nearly zero, and goes up to values where probability of localization is 1. For each SNR 50
trials were done, and then only those where the peak appeared at the exact location were considered as yielding the correct result.

First, and as a consequence of the requirement of the coherent processor, the data model in (3.4) was considered. This is equivalent to the model in (3.5) with \( p(\omega) = 1 \) with probability 1. The plots (a), (b) and (c) of figure 3.4 show the results obtained with this model for a number of frequencies of 4, 7 and 16 respectively. It can be noticed that as the number of frequencies increases the advantage of using the coherent (continuous line) over the incoherent processor (dashed line) is also increasing. This illustrates what is said in first paragraph of this section, about the rejection of noise by the coherent processor.

For figs. 3.4(d) and 3.4(e) the number of frequencies is constant and equals to 16, while \( p(\omega) \) has absolute value equal 1 and random phase with normal distribution, zero mean and variance 0.01. In fig. 3.4(d) different realizations of \( p(\omega) \) have highly correlated phases and in fig. 3.4(e) different realizations of \( p(\omega) \) have uncorrelated phases. The performance of both processors falls considerably when compared to 3.4(c). Being in the presence of a random signal should not have important impact on the incoherent processor’s performance, but the covariance matrices were sample covariance matrices, thus having not exactly the structure computed theoretically, and therefore changing the incoherent processor’s performance. The difference between (d) and (e) is negligible suggesting that the degree of correlation of the phase term in \( p(\omega) \) does not play any role in the performance evaluation of these two processors.

Finally, the simulation was carried out using a uniformly distributed phase (fig. 3.4 (f)) with 16 frequencies. This phase distribution was obtained modeling the real and imaginary parts of \( p(\omega) \) as normal distributed random variables. Thus, in this case the absolute value
Figure 3.4: Probability of correct source localization obtained for the incoherent processor (dashed) and the matched-phase processor (continuous) under different conditions (synthetic data): (a) 4 frequencies and $p(\omega) = 1$; (b) 7 frequencies and $p(\omega) = 1$; (c) 16 frequencies and $p(\omega) = 1$; (d) 16 frequencies and correlated phase of $p(\omega)$ with normal distribution, zero mean and variance 0.01; (e) 16 frequencies and uncorrelated phase of $p(\omega)$ with normal distribution, zero mean and variance 0.01; (f) 16 frequencies, phase of $p(\omega)$ uniformly distributed and amplitude normally distributed.
of $p(\omega)$ is no longer equal 1 and is a random variable too. The comparison shows that under these conditions the incoherent processor has higher ability to localize the source than the coherent processor. Attending to (3.8), this result suggests that the SNR obtained by the coherent processor is very low, forcing it to fail to localize more often than the incoherent processor. Note that these tests use sample matrices, hence the noise is not completely rejected.

It is also interesting to compare the structure of the ambiguity surfaces obtained by each of these processors (fig. 3.5). The structures are very different, since the coherent processor has considerably higher peak-to-sidelobe ratio. Another difference is that the resolution in range shown by the coherent processor is much higher than that of the incoherent processor.

![Figure 3.5: Examples of broadband ambiguity surfaces computed for frequencies 300, 400, 500 and 600 Hz and SNR equal 0 dB using the a) incoherent processor; b) matched-phase processor](image)

### 3.4 Environmental focalization

MFP is an array processing technique that is a generalization of plane-wave beamforming and uses the spatial complexity of acoustic field in the ocean to match replicas obtained by the solution of the Helmholtz equation with the field measured by an array of receivers. This technique was developed to perform source localization in the ocean in range, depth and
azimuth, provided that the environment of the propagation channel is known with sufficient accuracy. The amount of information available about the environment is a serious problem to deal with in MFP. The propagation model that solves the Helmholtz equation is fed with the parameters of the environment, and returns the replica field having into account those parameters and a given source position. To properly localize the source it is required to know the environmental parameters with enough accuracy. This is not always possible. Seafloor properties in shallow water are often characterized by strong variability, and the employment of seismic surveying and coring for exploring extensive areas is, in general, a very expensive and time consuming task. Another issue is time coherence of the environment. For example, if a source is to be located along time, or a moving source is to be tracked, important changes in the environment may occur over time. In fig. 3.6 the ADVENT’99 data set was used to show how acoustic data decorrelate over time for a period of 5 hours. The available LFM transmissions, comprising the band from 200 to 1600 Hz, were Fourier transformed and analyzed as follows: for each frequency a series of data segments received along time at the vertical array are compared to a reference segment. This is done by computing the absolute value of the internal vector product between the reference pressure vector and the other pressure vectors. As time elapses, the data becomes less correlated. This reflects changes in the environment - geoacoustic parameters, oceanographic parameters like temperatures, or internal waves do change in a periodic fashion. It can be observed that the field decorrelates faster and more at higher frequencies.

Focalization is a generalization of matched-field processing, in which both the source parameters and the environmental parameters are equally searched. Environmental focalization provides a powerful solution for the lack of accurate measures of the environmental
3.4. ENVIRONMENTAL FOCALIZATION

Figure 3.6: Matched-field Correlation of the received acoustic signals as a function of frequency and time. The data are LFM chirps acquired during the ADVENT’99 experiment for the 10 km track.

parameters, and to overcome mismatch.

An important concept that rises often in environmental focalization is "the equivalent model". Two models are equivalent if they are different in geometry, environmental parameters or range-dependence, while giving a similar acoustic response. For example, very few works were done under assumptions of range-dependence. In fact a range-independent environment does not exist. Over a few kilometers it is unlikely that bathymetry, seafloor properties, or even temperature in the water column are constant. The concept of equivalent models allows simplification of the modeling process by the use of an environmental model that has different parameters, while giving a similar acoustic response.

In some cases it is not possible to assume range-independence. This is shown by Gerstoft et al. [21] with real data acquired during the North Elba Sea Trial in October 1993 conducted by SACLANTCEN. Two data sets, with central frequencies of 170 and 335 Hz respectively, were available. Optimization of only geometric and geoacoustic parameters in a range-independent environment was found to be satisfactory at the lower frequencies, but for the higher frequencies optimization of additional parameters by inclusion of either a range
dependent bathymetry or ocean sound speed profile was essential for a successful inversion.

Another important concept is parameter hierarchy [12], that is the relative sensitivity of the acoustic field to the variation of a given parameter. In this hierarchy the parameters concerning the source location are on the top of the hierarchy. Since the main goal of MFP is source localization this hierarchy is fortunate. However, in certain cases the field is also very dependent on the waterdepth. The ambiguity between source range and water depth is a well known phenomena. As an example, fig. 3.7 shows a range-waterdepth ambiguity surface that illustrates this problem. The acoustic field was generated for a source at 10 km distance in a 80 m depth shallow water environment using a normal mode propagation model. Two frequency bands ranging from 200 to 700 Hz spaced by 100 Hz, and another for frequencies from 1000 to 1500 Hz spaced by 100 Hz were used. Then, range and water depth were scanned to obtain replicas to be matched with the simulated field using the conventional incoherent processor. It can be seen in fig. 3.7(a) that for the lower frequencies a high correlation band appears range-waterdepth pairs coincident with a straight line. This clearly shows that at these frequencies many solutions of the inverse problem produce equivalent frequency-responses. Observing in more detail it can be concluded that the relation between waterdepth estimates and range estimates is linear. In this example, 1 m in waterdepth corresponds to 0.5 km in range. At higher frequencies (fig. 3.7(a)) this effect still exists, but in a much more localized area in the bi-dimensional search space - the band turned into an ellipsis. One important issue is how unique the field is when compared to fields produced accounting for other source locations or environments. Low uniqueness may rapidly turn into a problem under environmental mismatch or noise. This example shows that for higher frequencies the uniqueness of field is higher, hence choosing an higher frequency set may
Figure 3.7: Range vs. waterdepth ambiguity surface obtained with the broadband conventional incoherent processor: (a) Lower frequency set (200-700 Hz); (b) Higher frequency set (1000-1500 Hz).

be part of the solution for the difficulty of unambiguously focusing the parameters in this situation. It looks like one should choose as higher frequencies as possible. But to work with high frequencies has a drawback, that is the difficulty in the modeling procedure, that can not be noticed here.
Chapter 4

Inverse Problems and Global Optimization using Genetic Algorithms

Determining the range and depth location of an acoustic source in a waveguide from the acoustic field measured on a vertical array of sensors can be seen as an inverse problem. The same applies when estimating the environmental parameters of a waveguide from the receiver acoustic field. Inverse problems are common to many areas of physics and functional analysis. Generic methodologies and algorithms are also applicable to our problem and will not be explained here in detail.

As a generic concept, inverse problems can be classified as well behaved or ill conditioned. Well behaved inverse problems are generally linear, analytical and allow a solution that is unique. In our case the derivation of the acoustic field in given environmental and genetic conditions is non-linear and non-analytical, moreover, since the received field is contaminated with noise there is no guaranty of uniqueness.

For such a ill conditioned inverse problem, the corresponding multi-dimensional objective function may exhibit several maxima among which the highest may not correspond to the true solution. In the last 10 years a number of techniques have been proposed in the literature
to cope with such optimization problems [39]. Among these techniques, Genetic Algorithm (GA) is a class of stochastic methods that have the following characteristics:

- perform global optimization;
- are able to escape from local extrema;
- have convergence to the true solution.

The GA is an optimization method based on principles of biological evolution of individuals [28]. An individual is a collection of bit chains that represents one of the possible parameter vectors, and a population is a set of individuals that evolves through time as generations. A generation is an iteration in which the fitness of each individual is computed by the so-called objective function. The fitness represents the "quality" of an individual. The probability of an individual to be included into the next generation depends on its fitness, i.e. individuals with higher fitness are more likely to survive. Two probabilistic operators are applied to the individuals: the crossover operator and the mutation operator. The crossover operator joins individuals into pairs without considering their fitness, and a given number of bits is exchanged with a given probability. The mutation operator inverts every bit with a given probability. This operator is important to avoid the loss of individuals’ variety. The loss of variety of individuals can lead to convergence to local extrema. Therefore the mutation probability should be set high enough to keep the search algorithm being able to escape from local maxima but low enough to not slowdown convergence to the global extremum, thus it is a compromise between speed and accuracy. At the beginning a random population of all possible vectors is selected. The fitness of each individual is computed. The operators crossover and mutation are applied to get a new population - the children. The fitness is
improved from generation to generation through evolutionary mechanisms. An evolutionary step consists of selection of individuals based on individuals’ fitness.

Global optimization will be applied to the matched-field procedure for the focalization of the geometry environment. The replica fields are obtained with the normal mode propagation model C-SNAP [15] feeding in the forward model parameters of the geometry and environment. The inverse problem corresponds to the estimation of the geometric and environmental parameters. To achieve this an objective function has to be evaluated, which is the conventional Bartlett processor that correlates the replicas with the measured field. This function has to be maximized in order to determine the estimated parameters. To achieve this maximization a GA implemented by Tobias Fassbender [14] has been used.

The GA is able to reach the maximum by sampling a very small number of points of the objective function which, in the case of MFP, allows the inversion of problems that would be unsolvable using traditional methods.
**Chapter 5**

**The ADVENT’99 sea trial**

**5.1 Experimental setup**

The SACLANT Undersea Research Centre and TNO-FEL conducted a sea trial in order to acquire acoustic data on the Adventure Bank off the southwest coast of Sicily (Italy) in May 1999. This area has a low range-dependence, and was found to be favorable for acoustic propagation. The data were acquired during the first three days of May. The acoustic source was mounted on a moored tower 4 m off the bottom at a depth of 76 m. The signals were received by an acoustic array of 64 elements. Each day it was deployed at different ranges of 2, 5, and 10 km. Broadband acoustic linear frequency modulated signals and multi-tones were transmitted every minute. To accomplish this, two sources were used, one for lower frequencies (200-700 Hz), and another for higher frequencies (800-1600 Hz). The transmission time was around 5 hours for the 2 and 5 km tracks, and 18 hours for the 10 km track.

For sound speed measurements a 49-element Conductivity-Temperature-Depth (CTD) chain towed by ITNS Ciclope with a data sampling every 2 s was used. The CTD chain spanned around 80% of the water column, and was towed continuously between the acoustic source and the vertical array (10 km track) during all transmissions in a parallel line to the
5.2 The baseline environmental model

The baseline model consisted of an ocean layer overlying a sediment layer and a bottom half space, assumed to be range independent, as shown on fig. 5.2. For the purposes of the inversion the forward model parameters were divided into four parameter subsets: geometric, sediment, bottom, and water sound speed. The geometric parameters included source range, source depth, receiver depth and bathymetry. The baseline sediment and bottom properties used for the ADVENT site were those estimated by Siderius [46] using the low frequency data (see table 5.1).

Fig. 5.3 shows an example of sound speed profile measured close to the array 5 km distance from the source at May 02, at 06:38. This is a typical double termocline profile at 10 m and 55 m depth with isovelocity layers in between.

The acoustic array had 64 receiving elements with different spacings, but throughout this work only the sensors spaced by 2 m are used giving a maximum of 31 sensors and an array
Figure 5.2: Baseline model for the ADVENT’99 experiment. All parameters are range independent. The model assumes the same density and attenuation for sediment and sub-bottom.

 aperture of 62 m (one sensor was used for another purpose).

Forward models were computed using the SACLANTCEN normal mode propagation model C-SNAP [15].

<table>
<thead>
<tr>
<th>Model parameter</th>
<th>Geometric</th>
<th>Sediment</th>
<th>Bottom</th>
</tr>
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<tbody>
<tr>
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<td>source range (km)</td>
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<td>comp. speed (m/s)</td>
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<tr>
<td></td>
<td>2, 5, 10</td>
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<td>1637</td>
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<td>source depth (m)</td>
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<td>density (g/cm³)</td>
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<td>array aperture (m)</td>
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<td>thickness (m)</td>
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<td>sensor spacing (m)</td>
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<td></td>
</tr>
<tr>
<td>array tilt (rad)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>water depth (m)</td>
<td>80</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1: Geometric and geoacoustic parameters used in the baseline model. Each day corresponds to a different source range.
Figure 5.3: Example of a sound speed profile measured at May 02, 1999 at 06:38.
Chapter 6

Experimental results: Source localization on the ADVENT’99 data

6.1 Data processing procedure

This section is to explain what is the general procedure applied on the data analysis for source localization on the three tracks in the next sections. The algorithm is divided in 2 steps:

- focalization of the geometry and environment;
- source localization.

The focalization of the environment and geometry (step 1) was posed as an optimization problem, that is to find a vector of parameters $\theta$ that maximizes an objective function, which in this case is the conventional incoherent processor obtained in (3.22). To successfully achieve the goal of this step a GA is used. The primary aim of this step is to determine the environment and parameters concerning the array. Therefore, the source location parameters are searched in narrow intervals.

Step 2 is range-depth source localization. The environment is fixed to the parameters obtained in step 1 and range-depth ambiguity surfaces are computed.
6.2. THE 2 KM TRACK

Throughout this work mostly the multi-tones will be used, which had the following frequencies:

- Lower frequencies comprising the interval ranging from 200 to 700 Hz, spaced by 100 Hz, i.e. 6 bins were used.

- Higher frequencies comprising the interval from 800 to 1500 Hz, where the bins were 800, 900, 1000, 1200, 1400 and 1500 Hz.

To compute the covariance matrix in (3.7), a single ping of 10 s was divided into 0.5 s segments, where the first and last segments were discarded, and then Fourier transformed. Thus, each covariance matrix is the average of 18 outer products.

6.2 The 2 km track

The first data to be analyzed is obviously that acquired on the 2 km track. The environmental parameters are those estimated for the 2 km lower frequency multi-tones by Siderius [46] and shown in table 5.2. The sound-speed profiles available for this day correspond to those measured close to the array (the times are shown in table 6.1). Since for this track the low-frequency has been already analyzed, only the higher-frequency data is going to be analyzed. In the focalization step were included only geometric parameters. Table 6.2 show the search boundaries and quantization steps taken when optimizing with the GA. The number of iterations was set to 40 with populations of 100 individuals. Crossover and mutation probabilities were set to 0.9 and 0.011 respectively. Note that all these values related to the GA parameters are canonical and had to be slightly re-adjusted for the optimizations in other ranges.
Table 6.1: Times (Julian times) at which CTD measurements were made closest to the vertical array.

### 6.2.1 Optimizing the geometry

The first trial in order to locate the source in the 2 km track, by looking for range and depth, using those geoacoustic estimates, assuming zero tilt and no receiver depth deviation failed. The reason could be the high sensitivity to geometric mismatch at high frequencies.

Then receiver depth and array tilt were estimated in a first focalization step (table 6.2 contains search bounds) keeping constant the geoacoustic parameters (see table 5.1). Note that the source and sensor depths were parametrized as the distance to the seafloor.

The bounds concerning the source location are very narrow: the goal of this step is fundamentally the estimation of the parameters concerning the array to be used for source localization. Thus, for this step, it is partially assumed that the source location is known. In general, the estimation results to be used for source localization are not going to be shown.

Finally ambiguity surfaces were computed for range between 1 and 3 km, and depth between 10 and 80 m (see figs. 6.1 and 6.2). The time elapsed between taken pings is 28 minutes except between surfaces 2 and 3 (where transmissions were interrupted). The position is quite constant, where only one outlier in range was obtained, which is a deviation of about 250 m relative to the expected range (2 km). The depth is well resolved between 76 and 77 m. The value of the peak shows some variability (fig. 6.3), where it can be seen that
6.2. THE 2 KM TRACK

<table>
<thead>
<tr>
<th>Model parameter</th>
<th>Lower bound</th>
<th>Upper bound</th>
<th>Quantization steps</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometric</strong></td>
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<td></td>
<td></td>
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<tr>
<td>Source range (km)</td>
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<td>2.6</td>
<td>128</td>
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<tr>
<td>Source depth (m)</td>
<td>1</td>
<td>10</td>
<td>128</td>
</tr>
<tr>
<td>Receiver depth (m)</td>
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<td>15</td>
<td>32</td>
</tr>
<tr>
<td>Tilt (rad)</td>
<td>-0.017</td>
<td>0.017</td>
<td>64</td>
</tr>
</tbody>
</table>

Table 6.2: GA forward model parameters with search bounds and quantization steps for the focalization step. Source depth and receiver depth are measured from the bottom.

The Bartlett power has a behavior that is monotonic over short time intervals. One could expect higher matched-field response for the acoustic data taken at instants close to those at which the sound-speed was. Considering the instants at which the sound speeds were taken, inversion 3 and 6 should show high values for the Bartlett power, but they do not. It is not known how well the geometry has been estimated, and whether there were other variations in the environment, or not.

A full inversion, optimizing all parameters in table 5.1, was then carried out in order to validate the geoacoustic parameters estimated in [46] using the low frequency data set. The search interval of each parameter is shown in table 6.3. The search space is of order $10^{17}$, which can be complicated to be searched even with a global optimization method, since high frequencies are involved, and may become computationally very intensive. Fig. 6.4 shows the results obtained. Concerning the geometry it can be noticed that for this range the ambiguity range-water depth is clearly present since both show the same variation. Also source depth and sensor depth have the same variation as range. Concerning the geoacoustic parameters, it can be seen that the field is sensitive only to the compressional speed in the sediment, and slightly to the density. All the other parameters have no (or little) effect on the measured acoustic field. This result shows that at these frequencies the geoacoustic parameters, except the speed in the sediment can be kept constant.
6.2. THE 2 KM TRACK

Figure 6.1: Ambiguity surfaces obtained for the 2 km track using the higher frequency multi-tones (between 800 and 1500 Hz) comprising an acquisition time of 8.5 hours. The geometric parameters were estimated in a previous step and the geoacoustic parameters are those in table 5.1.
Figure 6.2: Source localization results over time for the 2 km track using the higher frequency multi-tones (800 to 1500 Hz): (a) range; (b) depth.

Figure 6.3: Bartlett power of the surface main peak over time for the 2 km track using the higher frequency multi-tones.

<table>
<thead>
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<th>Model parameter</th>
<th>Lower bound</th>
<th>Upper bound</th>
<th>Quantization steps</th>
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<td>source range (m)</td>
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</tr>
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<td>water depth (m)</td>
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<td>84</td>
<td>64</td>
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<tr>
<td>tilt (rad)</td>
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<td><strong>Sediment</strong></td>
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<tr>
<td>thickness (m)</td>
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<td>10</td>
<td>64</td>
</tr>
<tr>
<td><strong>Bottom</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>comp. speed (m/s)</td>
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<td>250</td>
<td>64</td>
</tr>
</tbody>
</table>

Table 6.3: GA forward model parameters with search bounds and quantization steps.
Figure 6.4: Results of optimization carried out using the higher-frequency multi-tones (800-1500 Hz) in the 2 km track.
6.3 The 5 km track

6.3.1 Low frequency multi-tones

Having obtained stable results on the 2 km data set, the next step is to study the effect of a larger source range. But first, optimizations on the lower frequency data set were carried out. The geometric parameters and the water depth were searched, while the geoacoustic parameters were kept equal and fixed to those estimated in [46]. The sound-speed profile is chosen as an interpolation using sound-speeds measured close to the array (see table 6.1). Ambiguity surfaces computed with the optimized parameters are shown in fig. 6.5. The main peak is well resolved in both range and depth. An higher ambiguity appears at 50 m depth for the ambiguity surfaces from 3 to 7. These ambiguity surfaces are an important reference for source localization with the higher frequency multi-tones.
Figure 6.5: Ambiguity surfaces obtained with the broadband conventional incoherent processor for the 5 km track using the lower frequency multi-tones (200 to 700 Hz) taken in a time interval of 6 hours.
6.3.2 High frequency tones: blind focalization of main parameters

The first optimizations on the 5 km acoustic data using higher frequencies were carried out optimizing all parameters to which the acoustic field has shown to be sensitive in the previous optimizations on the 2 km range. Those are the geometric parameters (range, depth, array depth, array tilt), water depth, and sediment sound-speed. Every 32 minutes a 10 seconds ping was taken giving a total of 12 pings comprising nearly 6 hours. Fig. 6.6 shows the ambiguity surfaces obtained with parameter estimates obtained in a previous step (not shown). Significant ambiguities appear in every range-depth ambiguity surface. There is one ambiguity that appears in the first 6 surfaces close to the 75 m depth and between 5 and 6 km range. This gives hope regarding to an improved modeling scheme. In surfaces 4 and 5, the maximum is present on the same location which gives some confidence to correspond to the correct value, since they differ about 32 minutes in time. In surfaces 7 to 12 the structure that could be found in surfaces 1 to 6 completely disappears. As it can be seen, serious problems are present for the 5 km track.
Figure 6.6: Ambiguity surfaces obtained with the broadband conventional incoherent processor for the 5 km track using the higher frequency multi-tones (800 to 1500 Hz) taken in a time interval of 6 hours.
6.3.3 High-frequency tones: A first approach for sound-speed profile focalization

In the previous subsection serious problems for source localization were found. Having in mind that the wavelength is small, and most of the energy is propagated in the watercolumn, degrees of freedom can be given in order to improve the watercolumn modeling. It is possible to parameterize the sound-speed profile using one or several parameters that can be searched as free parameters and combined with measures made at site.

At sea it is impossible to keep track of the exact depth of most of the devices that are deployed into the water. This happens also with the CTD chain. The first and simple idea, regarding the watercolumn, is to assume that the sound speed profiles are known with a depth accuracy of 4 m, and look for the offset between the measured depths and the real depths.

The waterdepth and the sound-speed in the sediment were kept constant according to the baseline model. The values of sound speed profile offset in depth were estimated between $-3.5$ and $3.5$ m. The sound speed at the source depth is not known. The model may be trying to shift the sound-speed depths in such a way that the sound speed obtained at the source depth works as a reasonable estimate at that depth. The inclusion of this parameter into the search space did not have significant impact on the estimates of the geometric parameters relative to the previous optimization.

Fig. 6.7 shows the ambiguity surfaces computed using the optimized parameters. The improvement is dramatic, when compared to fig. 6.6. Now the source appears close to the correct location, and the ambiguities are reduced. The first three ambiguity surfaces do not agree with the results obtained for the lower frequency multi-tones. All the others from (4) to
(12) show similar results, having the source at 5.4 km in most of the cases. However, a strong ambiguity structure is always present. It can be noticed that the mainlobes and sidelobes are very large for these frequencies, and compared to those lobes in the ambiguity surfaces obtained for the lower frequencies. The reason may be a poorly focused environment. In the next section estimates of the sound speed at the source depth will be presented.
6.3. **THE 5 KM TRACK**

Figure 6.7: Ambiguity surfaces obtained with the broadband conventional incoherent processor for the 5 km track using the higher frequency multi-tones (800 to 1500 Hz). In a previous step geometry and sound speed profile depth were focused.
6.3.4 High-frequency tones: estimating the sound-speed gradient

During the experiment the temperature in the water column was measured continuously, but only down to 67 m depth. Thus, the next problem is to estimate the sound speed profile below this depth which is possibly very important for predicting the acoustic field since the sound source was located near the bottom at 76 m depth. Up to now the sound speed profile was completed by extrapolating the two deepest sound speed values down to the bottom.

The simplest approach is to assume that the sound-speed behaves linearly as the depth increases and estimate its gradient. Even if this assumption is too simple, it may be realistic since the sound speed typically behaves linearly with depth below the termocline.

Fig. 6.8 shows the sound speed profiles obtained after estimating of the gradient below 67 m depth. In general there is a strong change in direction at the bottom. It seems to be surprising, but observing the CTD in fig. 5.3, it is seen that below 50 m a strong decrease of sound speed (negative gradient) is present, and then around 70 m the gradient is positive. Hence there is some agreement between the real data and the estimates. Fig. 6.9 shows the ambiguity surfaces obtained with these parameters. The first three were greatly improved: the ambiguities were reduced and for the first and second surface the position of the main peak was changed to the expected location. In surfaces 7 to 12 more ambiguities are noticed, which is consistent with the results obtained for the lower frequency data set.

The Bartlett power is improved relative to the previous focusing, where some values were found close to 0.6. In general all values are higher (fig. 6.10).
Figure 6.8: Optimization results for sound speed gradients below 67 m depth for the 5 km track by focalization using the higher frequency multi-tones (800 to 1500 Hz).
6.3. THE 5 KM TRACK

Figure 6.9: Ambiguity surfaces for the 5 km track using the higher frequency multi-tones (800 to 1500 Hz). The focalization step included estimation of the gradient of the non-measured sound speed portion.
6.3.5 High-frequency tones: using empirical orthonormal functions to optimize the sound-speed profile

In order to obtain a better fit, optimizations on the sound speed profile can be carried out. The direct estimation of the sound speed profile is the simplest approach as it directly reflects the parameters required, however, in general a sound speed profile contains a large number of data points, thus direct estimation of those data points could be cumbersome. The ocean sound speed can be efficiently represented via shape functions. Shape functions are used to reduce the number of parameters to be estimated. One method that has been used extensively for ocean sound speed estimation is one based on empirical orthonormal functions (EOF). EOFs are shape function [20] that can be obtained from a database and are very efficient to reduce the number of data points. If historical data is available, an efficient parameterization in terms of EOFs leads to faster convergence and higher uniqueness in the optimal solution since a lot of information is already available and the search therefore is started close to the solution. For this purpose, for example, EOFs are constructed from representative data by sampling the depth dependence of the ocean sound speed. The EOFs
are obtained by using singular value decomposition (SVD) of a matrix $\mathbf{C}$ with columns

$$
\mathbf{C}_i = \mathbf{c}_i - \bar{\mathbf{c}},
$$

(6.1)

where $\mathbf{c}_i$ are the real profiles available, and $\bar{\mathbf{c}}$ is the average profile. The SVD is known to be

$$
\mathbf{C} = \mathbf{UDV},
$$

(6.2)

where $\mathbf{D}$ is a diagonal matrix with the singular values, and $\mathbf{U}$ is a matrix with orthogonal columns, which are used as the EOFs. The sound-speed profile is obtained by

$$
\mathbf{C}_{EOF} = \bar{\mathbf{c}} + \sum_{n=1}^{N} \alpha_n \mathbf{U}_n,
$$

(6.3)

where $N$ is the number of EOFs to be combined, which is selected by observation of the singular values by using some empirical criteria. Experimental results have shown that usually the 2 or 3 first EOFs are enough to achieve a high degree of accuracy. The use of EOFs involves historical data that in the case of the water column sound speed profile can be acquired over time and space. Thus, one can expect to have sufficient information to enable the model to obtain a profile that best represents the watercolumn over range, depth and time.

EOFs were used to improve the environmental focalization and hence the source localization results obtained up to now. The criteria used to select the number of relevant EOFs for the available data was

$$
\hat{N} = \min_N \left\{ \frac{\sum_{n=1}^{N} \lambda_n^2}{\sum_{m=1}^{M} \lambda_m^2} > 0.8 \right\}
$$

(6.4)

where the $\lambda_n$ are the singular values obtained by the SVD, $M$ is the total number of singular values, provided that $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_M$. For this data set the criteria (6.4) yielded $N = 3$, i.e. the first three EOFs are sufficient to model the sound speed with enough accuracy.
The coefficients $\alpha_n$, which are the coefficients of the linear combination of EOFs, are now part of the search space, i.e., they are searched as free parameters, such as the geometry and the other parameters concerning the sound speed. By trial and error, the search interval for these coefficients was set between -5 and 5. Fig. 6.11 shows comparisons between the sound speed profiles obtained by interpolation and those obtained through the combination of the EOFs. It can be seen that some are closely matched, but other differ over the whole watercolumn. This depends on how range or time dependent the sound speed was through the sea trial, or it was simply the sound-speed through which the best match between replicas and real data could be obtained. Observing the ambiguity surfaces obtained with the estimated sound-speed profiles (fig. 6.11), it can be seen that the main peak is always nearly at the same location, with negligible variations, and the ambiguities could be suppressed (fig. 6.12). Finally the plots shown in figs. 6.13 and 6.14 respectively summarize the localizations and Bartlett power obtained over time. The estimation of the sound speed by EOFs improved significantly the Bartlett power values relative to those obtained in section 6.3.4.
Figure 6.11: Comparing sound speeds obtained by interpolation of real sound speeds with sound speeds estimated through EOFs using the higher frequency multi-tones (800 to 1500 Hz) at the 5 km track.
Figure 6.12: Ambiguity surfaces for the 5 km track using the higher frequency multi-tones (800 to 1500 Hz). The focalization step included estimation of the sound speed profiles via EOFs.
6.3. THE 5 KM TRACK

Figure 6.13: Source localization results over time for the 5 km track using the higher frequency multi-tones (800 to 1500 Hz): (a) range; (b) depth.

Figure 6.14: Bartlett Power over time for the 5 km track using the higher frequency multi-tones (800 to 1500 Hz). The focalization step included estimation of the sound speed profiles via EOFs.
6.4 The 10 km track

The strategy for environmental focalization followed in section 6.3 has shown to be effective for the 5 km track, i.e., the model has shown to be matched with the reality in terms of acoustic field. It was also shown that the modeling scheme applied at 2 km fails to accurately localize the source at 5 km for the higher frequency data. Now the question is whether the focusing method applied at 5 km works also at 10 km.

The sound speed profiles available were all taken along the 10 km track (see fig. 5.1). In order to obtain the EOFs, a database of sound speed profiles was constructed with profiles updated every 5 minutes and comprising the whole 10 km track acquisition period. By applying criteria (6.4) it was found that 3 EOFs were enough to represent the temperature field.

As it was done for the 5 km track, one data ping acquired each 32 minutes comprising a period of almost 6 hours were taken. First focalization and source localization with the lower frequency multi-tones was carried out in order to obtain a reference. At 10 km range, it was supposed to be likely to have some variations in water depth along the transmission track. Thus, during focalization, water depth was included in the search space to assure that the correct values for this parameter is used in the source localization step. Fig. 6.15 shows plots with estimates of source range and water depth over time for the lower frequency multi-tones. The ambiguity between source range and water depth is clearly present, as it can be seen by observing the shape of the curves. Taking fig. 3.7 (a) as an example it can be seen that for a continuous set of source range and water depth a matched-field response close to maximum is obtained (note that this is noise free simulated data). If noise or a mismatch situation occurs, then the maximum of the objective function may change to another location on the
ambiguity surface. Furthermore in the case of a global search algorithm like a GA, very few points of the objective function are sampled. Hence it may be difficult for the algorithm to distinguish the solution with best response even without noise or mismatch. In this case it was not possible to obtain consistent estimates of the water depth, thus it was kept fixed.

Ambiguity surfaces in fig. 6.16 show that the source location is well resolved over the whole time series, and the main peak is well above over the ambiguities. The offset in range is more than 600 m relative to the expected value (fig. 6.17)a), and the depth estimates show a variability of 3 m 6.17)b). The Bartlett power over time is greater than 0.6, which means that the agreement between the model and the measured data is good (fig. 6.18).

Then the algorithm has been applied to the higher frequency band. Having in mind the ambiguity surfaces of fig. 3.7 and that the offset in range obtained for the lower frequency multi-tones was over 600 m, the waterdepth was included in the focalization in order reduce the offset in range by giving one more degree of freedom. Comparing the plots for source range and waterdepth (not shown), the dependence between source range and water depth
Figure 6.16: Ambiguity surfaces obtained with the broadband conventional incoherent processor for the 10 km track using the lower frequency multi-tones (200 to 700 Hz) taken a time interval of 6 hours. 5.1.
6.4. THE 10 KM TRACK

Figure 6.17: Source localization results over time for the 10 km track using the lower frequency multi-tones (200 to 700 Hz): (a) range; (b) depth.

Figure 6.18: Bartlett Power over time for the 10 km track using the higher frequency multi-tones (200 to 700 Hz).
is not noticed anymore, which is in agreement with the example in figure 3.7. The ambiguity surfaces obtained for the data pings analyzed are shown in fig. 6.19. It can be noticed that the main peak is not completely stable in its correct position, but it always appears close to the correct location. Range varies between 10 and 11 km (fig. 6.20a)), and the depth has a variability between 71 and 77 m (fig. 6.20b)). The Bartlett power is low, ranging from 0.3 to 0.5 (fig. 6.21), but the main peak was not overtaken by the sidelobes.

The source localization experimental results obtained for the multi-tones are satisfactory. However, considerable variation both in range and in depth is noticed. These tones comprised 6 frequencies ranging from 800 to 1500 Hz. The issue is to know whether the modeling has reached its limitations, or whether it is possible to increase the amount of information inserted in the localization process by using more frequencies. Experimental results have shown that more frequencies may result in more consistent localization results [47]. Thus the linear frequency modulated (LFM) signals collected during the sea trial were used in order to increase the number of frequencies to better focus the environment and the source location. A number of 15 frequencies ranging from 820 to 1500 Hz with a spacing of 23.4 Hz were used. However, only one signal realization was used in the averaging of the covariance matrices. Observing the ambiguity surfaces obtained (see fig. 6.22), and comparing with those obtained for the multi-tones 6.19, no obvious improvement has been obtained relative to sidelobe reduction. However fig. 6.23 shows that the variability in range over the whole estimation time has been reduced from 1 to 0.3 km if pings 6 and 12 are excluded. The depth has two outliers, but for the other estimates the variability is approximately the same as that obtain with tones. The Bartlett power is lower than for the multi-tones (fig. 6.24), and therefore model-data mismatch is higher in the best match situation.
Figure 6.19: Ambiguity surfaces obtained with the broadband conventional incoherent processor for the 10 km track using the higher frequency multi-tones (800 to 1500 Hz).
6.4. THE 10 KM TRACK

Figure 6.20: Source localization results over time for the 10 km track using the higher frequency multi-tones (800 to 1500 Hz): (a) range; (b) depth.

Figure 6.21: Bartlett Power over time for the 10 km track using the higher frequency multi-tones (800 to 1500 Hz).
Figure 6.22: Ambiguity surfaces obtained with the broadband conventional incoherent processor for the 10 km track using the higher frequency LFM signals (820 to 1500 Hz, spaced by 23.4 Hz).
Figure 6.23: Source localization results over time for the 10 km track using the higher frequency LFM signals (820 to 1500 Hz, spaced by 23.4 Hz): (a) range; (b) depth.

Figure 6.24: Bartlett Power over time for the 10 km track using the higher frequency LFM signals (820 to 1500 Hz, spaced by 23.4 Hz).
6.5 The 5 km track: Shrinking the array aperture

After working with an array spanning around three quarters of the watercolumn, and having obtained good quality results at three different ranges, the goal now is to study the dependency of the MFP procedure on the array aperture. As it is known, as the aperture is decreased less spatial discrimination is obtained, hence resulting in higher sidelobes relative to the main peak. In other words, and in the case of a vertical array, if the array aperture is low the acoustic field being seen by the array is nearly a planewave. Although the main peak-to-sidelobe ratio decreases with the array aperture, the main peak will theoretically always be at the correct position. However, in the case of real data, geometric and environmental mismatch is always present and the acoustic field is observed during a limited time. Thus it is expected that from a certain point the reduction of the array aperture will force the MFP procedure to fail to localize the source.

Reduced array aperture MFP was applied only to the 5 km track data, since for this range good results with the full array were available, and could be used as reference.

The first idea was to check how long the MFP procedure would be able to yield reasonable localization results for the 5 km track using the higher frequency multi-tones as the array aperture was decreased. The array configuration used up to now had 31 elements spaced by 2 m. The procedure to shorten the array aperture is to leave out sensors from the top and the bottom in such a way that the array is always centered in the watercolumn, keeping the same spacing between sensors. The environment and geometry used were those obtained in the last focalization step for this range, hence search being carried out only for range and depth.

Figure 6.25 shows the results obtained by reducing the array aperture by successive
6.5. THE 5 KM TRACK: SHRINKING THE ARRAY APERTURE

factors of 2, i.e., from the full 31 sensors to 16, then 8 and finally to 4 sensors. The black curve shows the result obtained previously with the full aperture. Comparing the curves for range and depth obtained with half aperture (blue) to those obtained with full aperture, only negligible difference is noticed. For 8 sensors (14 m of array aperture) it is still possible to obtain reasonable source localization results (red curves). Excluding the outliers in pings 1, 2, 7 and 10, the localization results obtained, both in range and depth are the same as previously. The 4 sensors aperture (6 m) represents a sampling of about 8% of the watercolumn. The degradation of the results by reducing the aperture to 4 sensors (green) is evident in fig. 6.25. Notice that the Bartlett power is highest for the 4 sensor configuration over the whole time even if the MFP procedure failed. The increment of the Bartlett power has been noticed since the aperture started to be reduced due to reduced complexity of the acoustic field seen by the array. The matched-field response is a measure of the similarity of the real acoustic field with the replicas produced by the numerical propagation model fed with different source locations. If the aperture is less, then the uniqueness of the acoustic field along the portion of watercolumn spanned by the field is lower, hence the match is higher.

There is the possibility of compensating the lack of aperture by the addition of a large number of frequencies. Taking the LFM signals, a continuous set of frequencies is available. 30 equally spaced frequencies between 800 and 1500 Hz were taken. This allowed more ambiguity surface averaging. Fig. 6.27 shows the improvement obtained. There are still 3 outliers and some variability, but now it is possible to guess that 5.4 or 5.5 km may be the correct value for range. This case shows that it was possible to partially compensate the lack of information obtained by spatial sampling by increasing the number of frequencies.
6.5. THE 5 KM TRACK: SHRINKING THE ARRAY APERTURE

Figure 6.25: Source localization results over time for different array apertures using the higher frequency multi-tones (800 to 1500 Hz): (a) Range; (b) Depth. Each color stands for a different number of sensors: black for 31; blue for 16; red for 8; green for 4.

Figure 6.26: Bartlett Power over time for different array apertures using the higher frequency multi-tones (800 to 1500 Hz). Each color stands for a different number of sensors: black for 31; blue for 16; red for 8; green for 4.

Figure 6.27: Source localization results over time for an array aperture of 6 m using the higher frequency LFM signals (820 to 1500 Hz, spaced by 23.4 Hz): (a) Range; (b) Depth.
6.6 Testing the matched-phase coherent processor on real data (5 km track)

Source localization with synthetic data carried out in section 3.4 has shown that it is advantageous to use the matched-phase processor under low SNR conditions. It would be interesting to make the comparison between incoherent and coherent processors using real data, even if there is no real acoustic data with low SNR available.

Equation 3.28 states that the peak value is dependent on the correlation between two frequencies. It is expected that signals at close frequencies are higher correlated than signals at well separated frequencies. Thus, it is expected to obtain higher matched-field response if one selects frequencies that are close to each other than that obtain if frequencies are well apart. On the other hand, if frequencies are well apart more information is available, since the channel is excited at a higher variety of wavelengths and therefore more features of the channel are explored.

Figure 6.28: Bartlett Power over time for an array aperture of 6 m using the higher frequency LFM signals (820 to 1500 Hz, spaced by 23.4 Hz).

Finally the geometry was focalized with 4 sensors, which did not conduct to any improvement.
Source localization is carried out with 8 sensors and using 30 frequencies in the band from 820 to 1500 Hz taken from the LFM signals at the 5 km track and discretized with a resolution of 1.4 Hz. This case was chosen for studying the behaviour of the response of the matched-phase processor’s depending on the frequencies being selected. Fig. 6.29 shows which frequencies were selected. In one case (circles) the frequencies are selected such that the spacing is constant (23.4 Hz), and in the other case frequencies are contiguously selected in groups of 5 (stars).

Fig. 6.30 shows the ambiguity surfaces obtained for the case where the frequencies are equally spaced. It can be seen that the sidelobe reduction is strong and the resolution in range is very high. The curves in fig. 6.31 show the source localization results and fig. 6.32 shows the power for each case. The circles correspond to the case where equally spaced frequencies were used, and the asterisks correspond to the case where frequencies were chosen in groups. It can be seen that the power obtained with the latter case is higher most of the time. Considering the result for the power obtained in (3.28), this experimental result effectively is in agreement with the theoretical result. Note that the covariance matrices are only of one
signal realization, therefore the variance of the estimator is high and the power estimates may be not always accurate enough to obtain a fair comparison.

Although the case using groups of frequencies yields higher results, the source localization results for the case with equally spaced frequencies resulted in better source localization, which may be related to the higher amount of information available if frequencies are selected well apart, as referenced at the beginning of this section.
Figure 6.30: Ambiguity surfaces obtained by the matched-phase coherent processor for the 5 km track using the higher frequency LFM (820 to 1500 Hz, spaced by 23.4 Hz).
6.7 Discussion

Acoustic field data and environmental data were collected in shallow water during the ADVENT’99 experiment conducted by the SACLANTCEN in May 1999 in Sicily. The source was fixed on a tower 4 meter above the seafloor, and the vertical receiver array was deployed at three different ranges in each day, 2, 5, and 10 km. The signals were transmissions of multi-tones and linear frequency modulation chirps, for two frequency bands, one in a lower
frequency band (200-700 Hz), another in a higher frequency band (800-1600 Hz).

First, unknown environmental or geometric parameters were focused using a genetic algorithm and using reduced search intervals for the location. Then, ambiguity surfaces were computed for source localization. Algorithms were first applied to lower frequency data to obtain reference results. This is the procedure in general steps.

For the 2 km track geoacoustic parameters estimated in a previous work with low frequency data sets were used and only the geometry (tilt, receiver depth) were estimated. This was applied only to the higher frequency multi-tones. Results are consistent and stable. Afterwards, a full inversion for the whole geoacoustic parameter set was carried out, and the results have shown that the only seafloor parameter that significantly contributed for the acoustic field was the sediment sound-speed.

The same algorithm was then applied to the 5 km track and allowed for perfectly stable results for the lower frequency data, but failed completely to localize the source for the higher frequency data. This is a direct consequence of the increase of the range. A larger range implies a higher number of parameters and their possible variability over time and space.

Since the wavelength is low (between 1 and 2 m) it is likely that most of the energy reaching the receivers is propagated through paths that have only few contacts with the boundaries or none at all, while the other are completely absorbed by the sediment. Thus, it is important to have a good knowledge about the water column.

Concerning the estimation of parameters in the water column, first it was assumed that the sound-speed depths were not accurate, thus a parameter representing a shift of the whole sound-speed was included. Estimating the uncertainty in sound-speed depths available gave a dramatic improvement: there was clearly a repetition of the structure shown by the
ambiguity surfaces with the peak appearing with slight shifts in range and depth around the correct location. As the temperature in the water column was not measured below 67 m, it was assumed that sound was linear with the depth from 67 m down to the water depth. Thus, the gradient of the sound-speed in this portion of water column was estimated, which brought stabilization to the source localization: only one outlier is present. The search for parameters to combine EOFs taken from historical data gave higher stability to the algorithm and helped to reduce or to eliminate ambiguities.

The whole process accomplished for the 5 km track at higher frequencies was repeated for the 10 km track, both for lower and higher frequencies multi-tones. For the low frequencies it resulted in high quality location estimates. However, for the high-frequency multi-tones some variability in range and higher in depth is noticed. In order to improve these results, more frequencies were used by taking the linear frequency modulated signals (30 bins between 800 and 1500 Hz). The environment and geometry were re-focused, some improvement in terms of variability was reached. However, these results indicate that for this environment and frequency range, a range of 10 km is already beyond the possibilities of a range independent environmental model. Surely waterdepth and probably the watercolumn should be modeled as range dependent.

The study concerning the shrinking of the array aperture has shown that with the higher frequency data set it is possible to obtain good results with an array spanning 15% of the watercolumn and reasonable results with an array spanning 8% of the watercolumn.

Finally, the coherent matched-phase processor was applied to the higher frequency LFM chirps using only 8 sensors. This test was carried out in order to study the main peak of the ambiguity surface. In one case frequencies were selected with constant spacing of
23.4 Hz, and in the other case with groups of 5 frequencies were selected. The results on source localization show that if frequencies are close then the main peak is higher than if frequencies selected with constant spacing. However, the localization results were better when frequencies were selected with constant spacing.
Chapter 7

Conclusion

Matched-Field Processing (MFP) is now a well known parameter estimation technique for localizing an acoustic source in range, depth and azimuth, using the spatial complexity of the acoustic field in the ocean. Among others there are two important aspects in MFP that are addressed in this work: one is the ability of a given processor to correctly pinpoint the source location by efficiently rejecting the sidelobes and the other is to study the impact of erroneous or missing environmental information on the final source location estimate.

One approach to increase sidelobe rejection is to combine information at different frequencies in a single broadband MFP estimate. There is a controversial issue in MFP on whether that information should be combined coherently or incoherently across frequency. The classical broadband MFP estimator is based on the incoherent averaging of the auto-frequency outer products. Coherent processors have recently been proposed, that use the average of the cross-frequency outer products weighted by a phase-shift term that represents a random phase difference between the two frequencies in the product. There is some trepidation in the literature about the rationale of that random phase term, that is classically represented as an uniformly distributed random variable in $[0,2\pi]$. In this work it was shown, using real data, that the distribution of the phase term at a given frequency approximately
follows a normal distribution. Moreover, it was also shown that if the observation noise is assumed to be uncorrelated across frequency than the cross-frequency outer products were not noise dependent. Instead, the limitations of the cross-frequency terms of the coherent processor where only dependent on the signal coherence along frequency, which seems to be quite variable from case to case. These developments where shown based on theoretical calculations and supported by simulation tests.

Concerning the broadband coherent processor important progress was made in its implementation. To look exhaustively for the phases conducted to prohibitive amounts of computation, since this meant to compute thousands of ambiguity surfaces, since the phases were searched as free parameters. By detailed analysis it was recognized that once computed the correlation terms the phases one is looking for are just the symmetric phases of the correlation terms. Thus one has only to apply this phase for all other points on the ambiguity surface. In [25] was mentioned that more than 3 frequencies would be computationally too expensive and global search algorithms had to be used to search for the appropriate phases. Thanks to the new approach, proposed here, it is possible to carry out tests with virtually any number frequencies, (cases with 30 frequencies have been tested) in very affordable computation times.

The second aspect referred to in this work was the analysis of MFP processors in presence of missing or erroneous environmental information. Since it is difficult to simulate real world mismatch situations this part of the work was based on a real data set acquired during the ADVENT’99 sea trial carried out by SACLANTCEN in the Strait of Sicily during May 1999. This data set had the particularity to be composed of three subsets of data at different ranges of 2, 5 and 10 km and at two frequency bands - a low frequency band between 200
and 700 Hz, and a high frequency band of 800 to 1600 Hz. The objective was to verify the consistency of the data along range and along frequency with emphasis on the high frequency band where extra environmental adjustment was expected to be necessary relative to the low frequency data. Another test was based on decreasing the vertical array aperture and search for the minimum array length giving accurate source localizations. An attempt was also made for compensating array aperture with frequency band. A two stage MFP algorithm was applied throughout the processing of the data: a first stage for environmental focalization using genetic algorithms to search for the parameters giving the best Bartlett fit and then a second stage for computing the MFP ambiguity surfaces in range and depth for source localization. Several conclusions can be drawn from this real data analysis: first, is that the bottom parameters could be estimated for the short range of 2 km and hold constant for all the other ranges up to 10 km; second, it was found that as the range was increased the water column variability was becoming more and more important to obtain correct matches, with particular difficulties for the high-frequency band; third, that water column variability could be modeled using an EOF expansion of the sound speed profile which coefficients could be estimated with the focalization process; fourth, precise MFP localizations could be obtained at all ranges and with both frequency bands. The attempt for decreasing the array aperture has showed that at 5 km source range, it was possible to achieve nearly correct localizations with an array 4 times smaller than the initial full aperture array, sampling only \( \frac{1}{6} \) of the water column. The result obtained with the short array was ameliorated by increasing the frequency band which actually allowed to reduce the array aperture even further to only 4 sensors (\( \frac{1}{12} \) of the water column).

As a final test the phase-coherent processor was used on the real data set in order to val-
idate the model assumption that coherence between frequencies in a neighborhood is higher than the coherence between frequencies selected well apart. It had been shown theoretically that the peak value depends on the correlation between the selected frequencies. The results has shown that the peak is higher when frequencies are selected in groups, but the source localization performance is better for frequencies selected well apart.
Bibliography


