GROWTH

1) Growth rates
2) Modelling growth

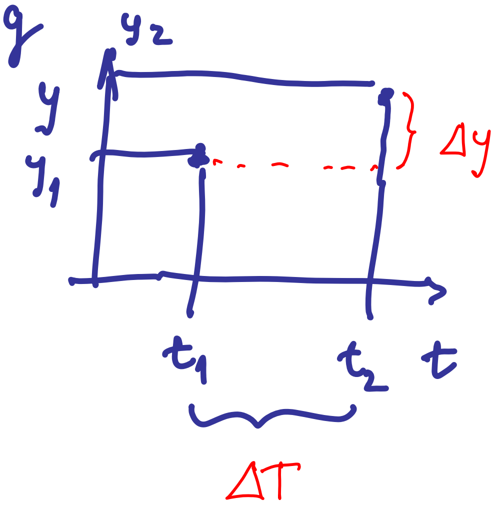
$\rightarrow \begin{cases} \text{length} \\ \text{weight} \end{cases}$

1) GROWTH RATES

a) Absolute average growth rate, tma
b) Relative average growth rate, tmr
c) Specific growth rate, g

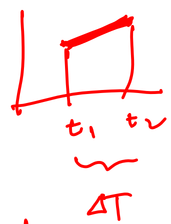
$y = \begin{cases} \text{length} \\ \text{or} \\ \text{weight} \end{cases}$

$$\underline{tma} = \frac{\Delta y}{\Delta T}$$



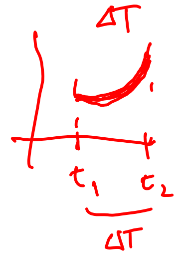
$$\underline{tmr} = \frac{1}{y^*} \cdot \frac{\Delta y}{\Delta T}$$

tma and tmr



Normally $y^* = y_1$

$$g = \frac{\ln y_2 - \ln y_1}{\Delta T} \rightarrow$$



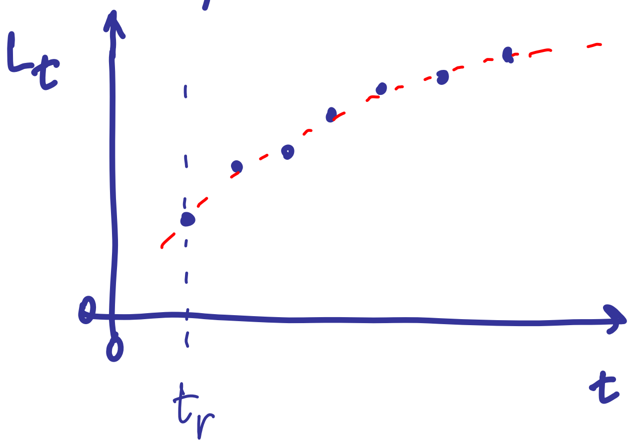
GROWTH MODELLING

Approach > To model growth in length and obtain the model in weight afterwards by using the length-weight relationship

Data

Mean length at age, L_t

t	L_t
t_1	L_1
t_2	L_2
t_3	L_3
$\vdots$	$\vdots$

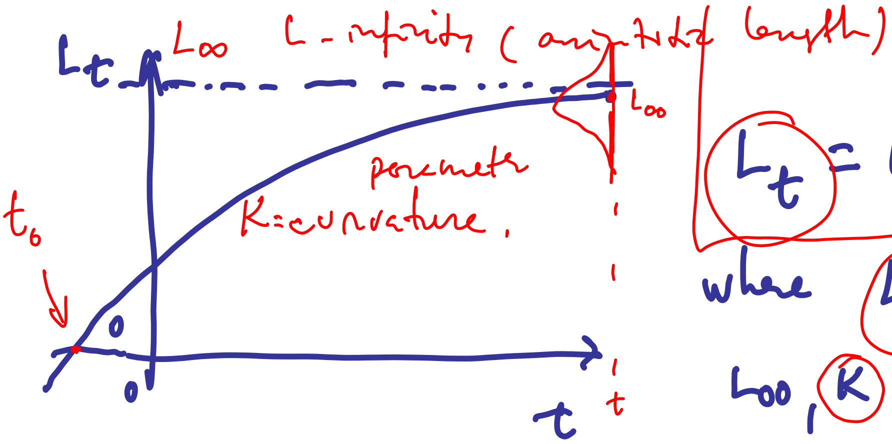


- age-length keys data
- backcalculated data
- length frequency Analysis data

MODELS to describe growth

Von BERTALANFFY growth curve

the most importante model, the most used to model marine species



$$\boxed{L_t = L_\infty \cdot \left[1 - e^{-K(t-t_0)}\right]}$$

where L_t, t are variables

L_∞, K, t_0 are parameters

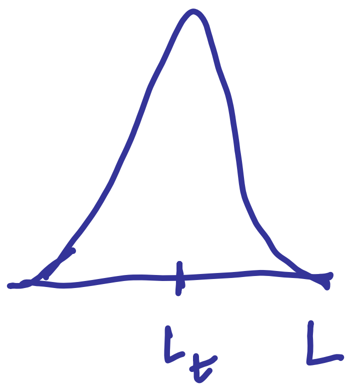
L_∞ = asimptotic length

K = curvature parameter or growth coefficient

t_0 = theoretic age at which $L_t = 0$

ex

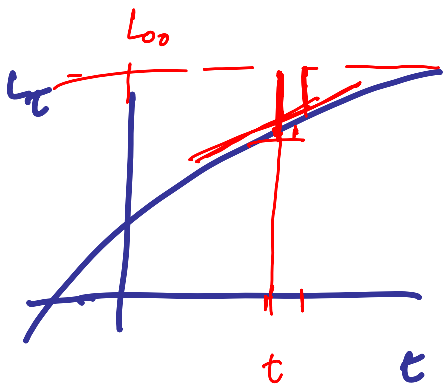
age t



Basic assumption

$$\frac{dL}{dt} = K\left(L_\infty - L_t\right)$$

$$K = \frac{1}{(L_\infty - L_t)}\frac{dL}{dT}$$



MODEL GROWTH IN WEIGHT

given a length-weight relationship

$$W = q \times L^{b}$$

q = allometric condition coeff.

b : allometric coefficient

and the model in length

$$L_t = L_\infty \left[1 - e^{-K(t - t_0)}\right]$$

$$W_\infty = q \, L_\infty^{\,b}$$

when

$b = 3$

$$W_t = W_\infty \cdot \left[1 - e^{-K(t - t_0)}\right]^3$$

Mean weight at age

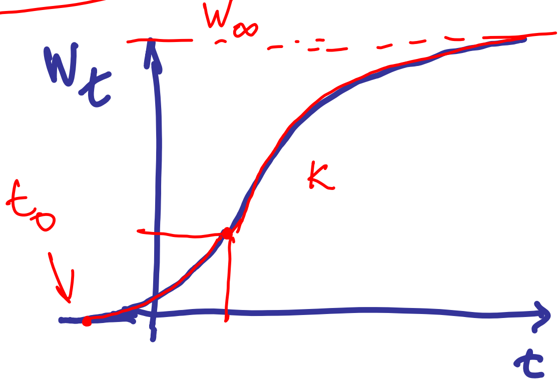
Von Bertalanffy growth curve in weight

$b \neq 3$

$$\boxed{W_t = W_\infty \cdot \left[1 - e^{-K(t - t_0)}\right]^{b}}$$

Richards growth curve (in weight)

graphic



CADIMA

SPARRE y VENEMA

$$W_{inflex} = \left(1 - \frac{1}{b}\right)^{b} \times W_{\infty}$$

$$t_{inflex} = t_0 - \frac{1}{K} \ln\left(\frac{1}{b}\right)$$

when $b = 3$

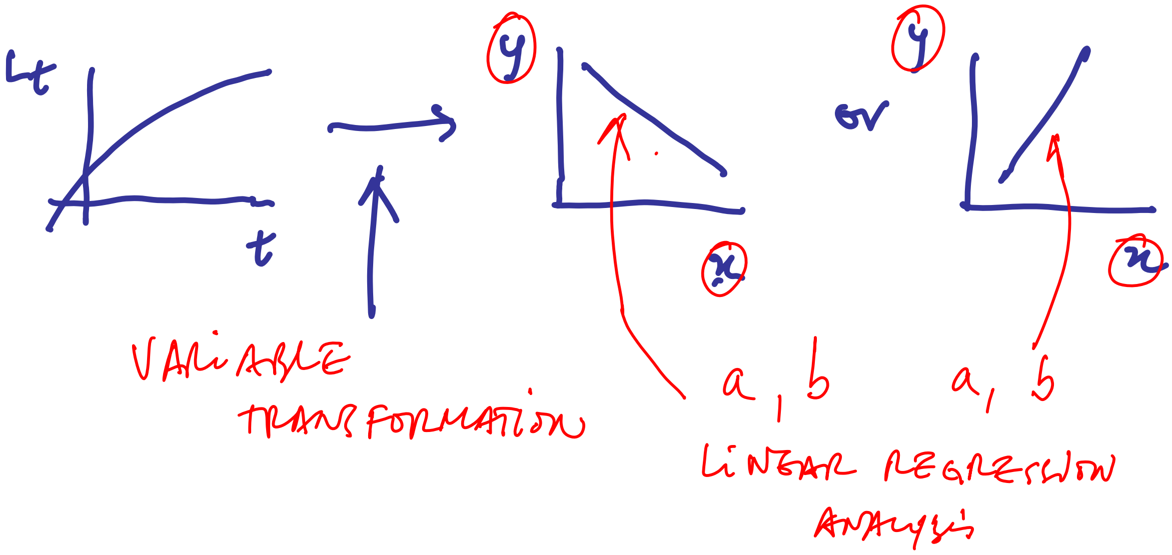
$$W_{inflex} = 0{,}2963 \cdot W_{\infty}$$

$$t_{inflex} = t_0 + \frac{1}{K} \ln 3 \quad \text{or} \quad \left(t_0 - \frac{1}{K} \ln\left(1/3\right)\right)$$

PARAMETER ESTIMATION

1) LINEAR ESTIMATION (GRAPHIC METHODS)

2) NON-LINEAR ESTIMATION (ITERATIVE METHODS)

1) 

K, L_∞ or given L_∞ K, t_0

can be obtained from a and b

intercept slope

of the regression line

GULLAND - HOLT PLOT

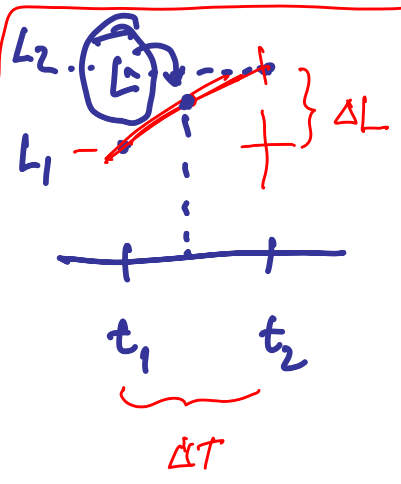
to estimate L_∞ and K

$$\frac{\Delta L}{\Delta T} = K\,(L_\infty - \bar{L}) \iff \boxed{\frac{\Delta L}{\Delta T} = K L_\infty - K \bar{L}}$$

where $\bar{L} = \dfrac{L_1 + L_2}{2}$

$\Delta L = L_2 - L_1$

$\Delta T = t_2 - t_1$



ΔT can be $\neq$ const

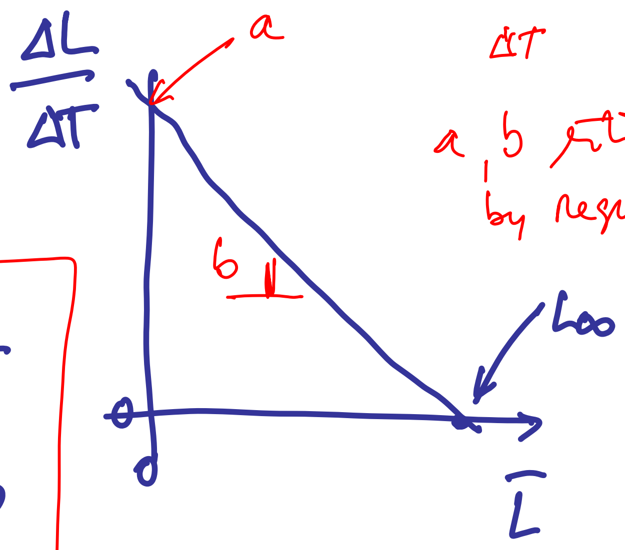
$$\frac{\Delta L}{\Delta T} = K L_\infty - K \bar{L}$$

$$y = a + b\,x$$

$a = K L_\infty$

$b = -K \therefore$

$$\boxed{L_\infty = -\frac{a}{b} \qquad K = -b}$$



a, b estimated by regression

FORD-WALFORD PLOT

$$L_{t+\Delta T} = L_\infty \left(1 - e^{-K\Delta T}\right) + e^{-K\Delta T} \times L_t$$

$$y = a + b \times x$$

$L_t = L_1$

$L_{t+\Delta T} = L_2$

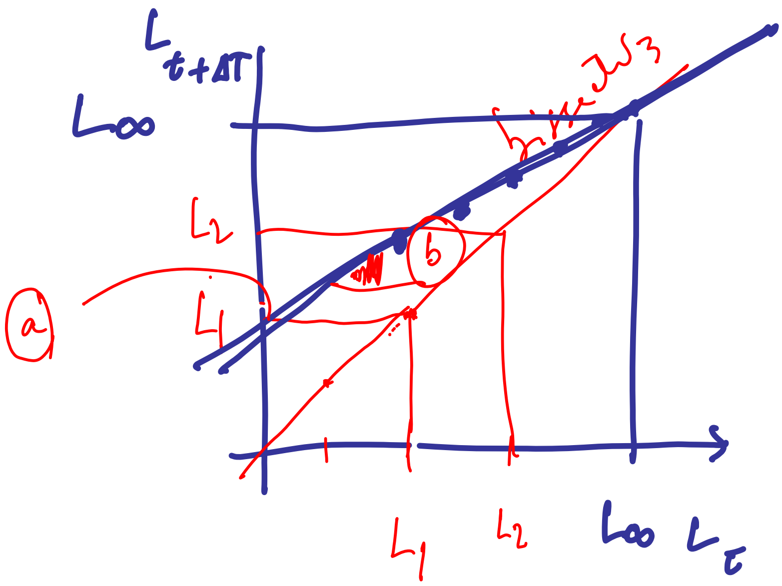
ΔT is constant

$$a = L_\infty \left(1 - e^{-K\Delta T}\right)$$

$$b = e^{-K\Delta T}$$

$$\therefore K = -\frac{1}{\Delta T} \ln b$$

$$L_\infty = \frac{a}{1-b}$$



VON BERTALANFFY PLOT

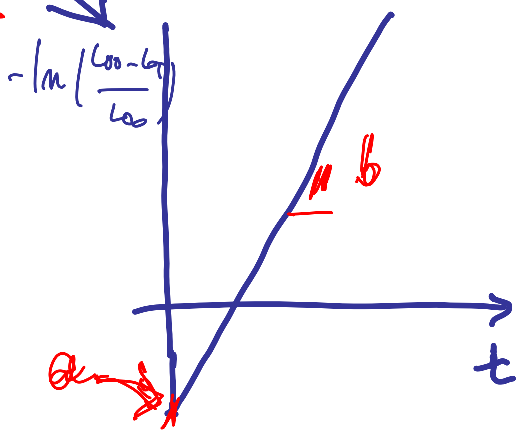
Given L_∞

to estimate K and t_0

$$\underbrace{-\ln\left(\frac{L_\infty - L_t}{L_\infty}\right)}_{y} = \underbrace{-Kt_0}_{a} + \underbrace{K}_{b}\,\underbrace{t}_{x}$$

$$a = -Kt_0 \qquad \boxed{t_0 = -\frac{a}{b}}$$

$$b = K \quad \therefore \quad \boxed{K = b}$$



Ex

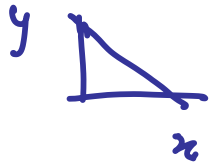
observed data

t	L_t
t_1	L_{t_1}
t_2	L_{t_2}
t_3	L_{t_3}
$\vdots$	$\vdots$

Gulland - Holt plot

y $\frac{\Delta L}{\Delta T}$	$\bar{L}$ x
$\frac{L_{t_2} - L_{t_1}}{t_2 - t_1}$	$\frac{L_{t_1} + L_{t_2}}{2}$
$\frac{L_{t_3} - L_{t_2}}{t_3 - t_2}$	$\frac{L_{t_2} + L_{t_3}}{2}$
$\vdots$	$\vdots$

plot



regression analysis

$$\left.\begin{matrix} a \\ b \end{matrix}\right\} L_\infty , K$$

r^2

n

observed data

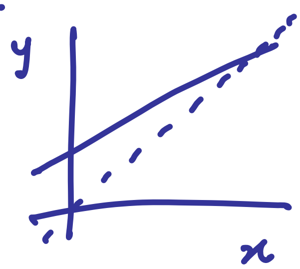
t	L_t
t_1	L_{t_1}
t_2	L_{t_2}
t_3	L_{t_3}
$\vdots$	$\vdots$

FORD - WALFORD

L_t	$L_{t+\Delta T}$
L_{t_1}	L_{t_2}
L_{t_2}	L_{t_3}
$\vdots$	$\vdots$

ΔT = const

plot



regression analysis

n

r^2

$$\left.\begin{matrix} a \\ b \end{matrix}\right\} K , L_0$$

BERTALANFFY PLOT

Ex

GIVEN

observed data

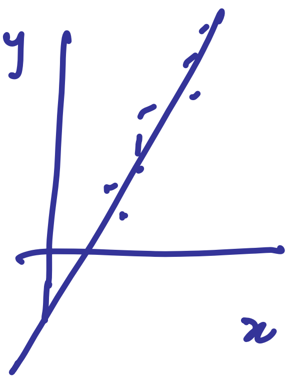
t	L_t
t_1	L_{t_1}
t_2	L_{t_2}
t_3	L_{t_3}
⋮	⋮

$-\ln\left(\dfrac{L_\infty - L_T}{L_\infty}\right)$	t
$-\ln\left(\dfrac{L_\infty - L_{t_1}}{L_\infty}\right)$	t_1
$-\ln\left(\dfrac{L_\infty - L_{t_2}}{L_\infty}\right)$	t_2
⋮	⋮

Mean point of the age group

$\downarrow$

plot
+
regression analysis



$\downarrow$

$$\left.\begin{matrix} a \\ b \end{matrix}\right\} K,\ t_0$$

n r^2

2) NON-LINEAR ESTIMATION (ITERATIVE METHODS)

EX

observed data

t	L_t
t_1	L_{t_1}
t_2	L_{t_2}
t_3	L_{t_3}
⋮	⋮

FROM GRAPHICAL METHODS

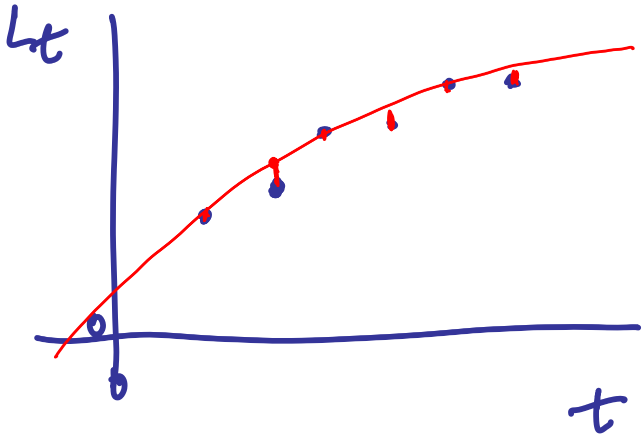
$L_{\infty}(0)$

$K(0)$

$t_0(0)$

Initial values for parameter

$$L_t = L_{\infty(0)}\left[1 - e^{-K_{(0)}(t - t_{0(0)})}\right]$$



CRITERIA TO OBTAIN THE BEST CURVE

$$\underset{SSR}{\phi} = \Sigma(\underbrace{obs - exp}_{\text{residual}})^2 = \underline{\underline{\text{MINIMUM!}}}$$

SSR

"least squares" criteria

WITH NUMERICAL ALGORITHMS COMPUTER FIND THROUGH ITERATIONS THE BEST COMBINATION OF THE 3 PARAMETERS THAT FITS <u>SSR MINIMUM</u>

EXCEL → SOLVER