Permeability fields estimation by conditional simulations of geophysical data

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Abstract A new method for the determination of permeability fields in highly heterogeneous aquifers is proposed. It stems from the known relations between soil electric resistivity and permeability. Several cut-offs are made to electric resistivity probability distribution. These cut-offs are heuristic estimates about what classes of values of one variable should relate to what classes of values of the other. The outcome of this process is a set of cumulative indicator variables. These variables are then simulated. The permeability fields are obtained by the intersection of the indicator variables. These permeability fields may be used in the development of conceptual models; autocorrelation distances (integer scales) obtained on the permeability fields may be used as input to macrodispersivity models. The method was applied to the Karst aquifer of the Escarpão, in central Algarve, Portugal. The images obtained fitted well the geologic structures identified by field works. The results of this method may be useful for the selection of new drilling spots, as input to flow and solute transport models, for the a priori determination of macrodispersivity parameters, and of the fractal behaviour of permeability distribution. This approach may be a very useful tool for aquifer parameterisation when the available information is scarce.

INTRODUCTION

Several studies incorporated indirect information to estimate hydraulic parameters of aquifers. Examples are: Ahmed et al. (1988), and Ahmed & Marsily (1987) who estimated transmissivity by cokriging with transversal resistivity; Copty et al. (1993) who used a Bayesian method to estimate permeabilities ($k$) through seismic waves propagation velocities; Schafmeister & Burger (1995) used indicators to estimate hydraulic conductivities ($K$); Schafmeister (1996) applied the indicator coding to estimate $K$ from lithologic data; Garcia & Froidevaux (1996) cokriged the indicators to generate contamination probability maps; Dimitrakopoulos & Dagbert (1993) and Muge et al. (1997) used sequential simulation of the indicator to generate permeability fields in a manner similar to the one proposed here, but with data only of direct nature (lithologic). We propose here a method of obtaining estimates of the permeability fields in highly heterogeneous media, fractured/karstified, to be used when hard data is scarce or non-available. The permeability ($k$) fields are estimated by conditional simulation of the indicator, which are dependent only on geophysical data where the apparent electric resistivity ($\rho_a$) is compulsory. The method is general for geophysical methods as long as they provide information about the number and
thickness of the lithologic layers and their electric resistivity. It was tested for Very Low Frequency-Resistivity. This method may be useful when the spatial distribution of the permeability is not known, nor its heterogeneity and isotropy. Karst systems are therefore good candidates. In the form presented here it should only be applied in cases where direct information isn’t available, nor other indirect data. If \( K \) or \( T \) values should be present then co-simulation should be used. This is a very small change in the method here described. The method was applied to a karst aquifer in the south of Portugal (Algarve): the Escarpão aquifer, in the area surrounding the Albufeira landfill.

**Relations between electrical resistivity and aquifer permeability**

Despite the fact that different physical laws describe electrical current and groundwater flow, some analogies have been established between them. Some authors determined empirical relations, linear or logarithmic, between hydraulic conductivity and apparent electrical resistivity \( K = f(\rho_w) \), or between transmissivity and transverse resistance, \( R \), \( [T = f(R)] \). \( R = \rho H \), where \( \rho \) is the formation electric resistivity, \( H \) is the aquifer thickness (or lithologic layer thickness). The longitudinal conductance, \( C_e \), is defined as \( C_e = \rho/H \), or \( C_e = \sigma H \), with \( \sigma \) the electrical conductivity. These relations are based on the relation between Darcy’s law Ohm’s law (equation 1) – see Table 1 – where \( Q \) is the yield and \( J \) the hydraulic gradient; \( A \) is the cross sectional area perpendicular to the flow; \( i \) is electrical current density; \( \sigma \) is the electrical conductivity \( \equiv 1/\rho \), and \( E \) is the potential. Ohm’s law relates the electrical current, difference of potential and the electrical resistivity, so that \( \partial V = \partial R \), and \( \partial R = (\rho \partial L)/ \partial A \), replacing comes \( \partial V/\partial L = -\rho I/\partial A \), where the left term represents the gradient through the element of length \( \partial L \), in \( V \ m^{-1} \); \( I \) is in \( A \ m^2 \).

Considering a prism of aquifer material with unit cross sectional area, and thickness \( H \), the two laws can be combined (Sri Nivas & Singhal, 1981) (equation 2). Maillet (1947) named \( R e C_e \) as Dar-Zarrouk variable and function, respectively, and form together the Dar-Zarrouk parameters. Other authors used an adimensional parameter obtained by the division of electrical resistivity, \( \rho \), by the water electrical resistivity, \( \rho_w \), giving rise to the formation factor, \( F \), (Archie (1942) (equation 3). That is, there is a relation between \( F \) and porosity, \( n \), with \( m \) and \( a \) constants related to the medium. Archie (1950) established the relation between permeability, \( k \), and porosity expressed by (equation 4).

Porosity may be calculated by the second equation in (3) and replaced in (4) to obtain a permeability estimate from the electrical resistivity of the medium and the fluid. This relation is fundamental for the development of many other models based on the correlation between \( n \) and \( \rho \) (e.g., Urish, 1981; Kosinski & Kelly, 1981); between \( \rho \) and \( K \) (e.g., Heigold et al., 1979); between \( R \) and \( T \) (Sri Nivas & Singhal, 1981, 1985); between \( F \) and \( k \) (Kwader, 1985). The Very Low Frequency – Resistivity is mentioned because it was used in the case study described latter in the text. The basis of the method can be found in most of the geophysical textbooks. The sensitivity and precision of the devices limited for some time the application of VLF-R methods to karst media. However, Müller (1982, 1983) showed encouraging results with devices developed specifically for ground water research. Further developments made them capable of operating in the 12 to 240 kHz, more suited for the purposes. In fissured, fractured and/or karstified formations the distinction
between altered (potentially more aquifer) and non-altered rock is usually possible with this method.

Table 1  Equations relating geophysical and hydrological parameters. See text for nomenclature.

<table>
<thead>
<tr>
<th>Equation</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
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<tbody>
<tr>
<td>( Q = K J A )</td>
<td>( i = \sigma E )</td>
<td>( T = K \sigma R )</td>
<td>( F = \frac{\rho}{\rho_w} )</td>
<td>( k = b n^c )</td>
</tr>
<tr>
<td>( T = \frac{K}{\sigma C_c} )</td>
<td>( F = a n^{-m} )</td>
<td>( b ) and ( c ) are constants</td>
<td></td>
<td></td>
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**Stochastic simulations**

In the Sequential indicator simulation (SIS) the original values are transformed into indicators, defined for established cut-off values (based on the distribution function or in any other relevant information). Conditional distributions are computed for each estimated node by simple kriging of the indicators. If indicator variables are not independent then disjunctive kriging should be used. Sequential methods have several advantages, especially when dealing with anisotropies and conditioning.

**ESTIMATION OF THE POTENTIAL PERMEABILITY FIELDS**

The methodology here proposed is should be applied only when no other data but geophysical data (phase and electrical resistivity) is available. The main objective is to estimate potential permeability fields in highly heterogeneous aquifers. The definition of potential permeability will be discussed latter. These fields may be used as input state variables into simulation models, or used to estimate dispersive parameters of aquifers (Nunes, 1998). The interpretation of apparent electrical resistivity \( (\rho_a) \) and phase \( (\varphi) \) pairs for different wave frequencies allows the estimation of the lithostratigraphic sequence and the real electrical resistivity \( (\rho) \) of each layer - inversion procedure. The method proposed considers models that estimate both real electrical resistivity \( (\rho(x)) \) and layer thickness \( (h(x)) \). The number of layers is a function of the phase inversions detected between different wave frequencies, and naturally of the skin depth (exploration depth). After the inversion of the geophysical data \( (\rho_a(x), \varphi(x), x=(x,y)) \), a matrix \( \Delta' (\Delta'_{ns} = \{ \rho(x_i,x_j,l), h(x_i,x_j,l) \}, x_i, x_j = \text{co-ordinates of sampled or estimated data points}, l = 1,..., \text{Number of lithofacies at the point}) \) of real electrical resistivities, \( \rho \), and layer thickness, \( h \). If layer thickness is divided in segments of equal dimension, a three-dimensional data set is obtained. So, FLF-R data set, in the form of apparent electrical resistivity and phase pairs for each wave frequency, is inverted with an inversion model (see, e.g., Fischer & Le Quang, 1981, or Kaufman & Keller, 1981), and new information about the number and thickness of the layers detected between the surface and a limiting depth - the skin depth -, and their real electrical resistivity is obtained in all the initial data points. By dividing the layers’ thickness into segments and assigning to their centre the real electrical resistivity value of the layer, then a depth property variation is added to the horizontal information. It is proposed that heuristic relations between \( \rho \) and \( \varphi \) be used. These relations are based on the modeller experience and in the physical phenomena under study. Direct
relations are made between intervals of electrical resistivity and intervals of potential permeability - that is relations between classes. N cut-off values are defined on the variable \( \rho \), \( \rho_1 \), \( \rho_2 \),..., \( \rho_N \), resulting N indicator variables \( I(x) \):

\[
I(x, \rho_i) = \begin{cases} 
1 & \text{if } \rho(x) \leq \rho_i \\
0 & \text{if } \rho(x) > \rho_i 
\end{cases}
\]

with \( i = 1..N \) cut-off values

One advantage of using cumulative indicator variables is that the indicator cokriging may no longer be necessary; other is emphasised by the method described above for determination of transition probabilities. To determine the transition probabilities experimental variograms for all the important directions in each indicator variable are determined, as well as the ratios between cross-variograms and simple variograms (Rivoirard, 1990, 1994). Indicator fields are estimated (by kriging or simulation) with the method that best fits the properties identified in the identification procedure. The outcome is N separate cumulative indicator fields. But we are interested in the potential permeability on each estimated point. With the N cut-off values, \( N+1 \) non-cumulative variables, \( I'(x) \), may be drawn: lower that the lowest cut-off, in between the former and the second lower cut-off, and so on until the last cut-off; and one variable with values higher than the highest cut-off. Each of the former variables is associated with a well defined electrical resistivity interval, and therefore represents a well defined potential permeability class (where \( k'_i(x) \) are potential permeabilities.:

\[
I'(x) = k'_i(x), \ i = 1,...,N+1
\]

APLICATION TO A CASE STUDY

The proposed methodology was applied to the karst Escarpão aquifer in the south of Portugal - Algarve region. The Escarpão formations are part of a sequence of limestones and dolomites of the Kimmeridgian medium-Portlandian, with average thickness of 700 m. They are highly fractured and karstified, specially in the more dolomitised areas, therefore with good hydraulic properties. The upper part of the sequence is less permeable. Groundwater flow is made essentially through the fractures and dissolution channels, with preferential development in the N20 and N140 directions (Almeida, 1985). Under this formation are the marls of the Peral that constitute the border of the aquifer.

The VLF-R profiles were made at 19, 77.5 and 183 kHz with sources direction of N30 with a device with a frequency range expanded up to 300 kHz. Profiles were made in the E-W direction, therefore favouring the detection of structures with a N-S alignment, but the vertical soundings may mask their influence. The spacing between sounding points was kept as close as possible to 25 m. Electrode spacing was constant and equal to 5 m (sampling precision). The field data collected were out of phase (phase), \( \varphi \), and apparent electrical resistivity, \( \rho_a \), in each of the frequencies. The following heuristic relations between \( \rho \) and \( k' \) were assumed: \( \rho < 55 \Omega \text{m} \Rightarrow \) high conductive fractures or karst channels (very high potential permeability); \( 55 < \rho < 110 \Omega \text{m} \Rightarrow \) less conductive fractures and karst conduits - smaller dimensions (high permeability); \( 110 < \rho < 220 \Omega \text{m} \Rightarrow \) small fractures in the blocks (low permeability); \( \rho > 220 \Omega \text{m} \Rightarrow \) the blocks - unaltered rock (very low
permeability). The classification was therefore: $k_1^p$: Very high permeability; $k_2^p$: High permeability; $k_3^p$: Low permeability; $k_4^p$: Very low permeability. These classes of permeabilities are represented in Figure 1 as white, light grey, dark grey and black. The areas of higher permeability are in lighter colours, allowing a fast visual interpretation of the potential location of groundwater carrying conduits.

**Results**

The results are in good agreement with the known geology of the area and are therefore very encouraging for further developments and testing. Figure 1 shows five vertical E-W cuts along the N-S direction, between the E-W co-ordinates UTM 571000 and 571900, and N-S UTM 4099640 and 4111110. The depth is constant and equal to 40 meters below surface. The images were obtained from a three-dimensional field. The central area of Figure 1 represents a more conductive N-S channel expected from the known geology of the area. This channel is in fact composed of several areas of higher permeability intermingled in lower conductivity materials as a result of strong fracturation and karstification.

These results are very encouraging as to the practical application of such a method to obtain permeability fields from geophysical data. Future developments must include the crossed analysis of other variables (in the framework of disjunctive kriging). The other variables may include, e.g., geophysical data, K or T, h, and water quality parameters. Stochastic simulations may generate several three-dimensional potential permeability fields, used as input for the definition of conceptual models. It is therefore easier to estimate water movement and contaminant transport in areas where the available hydrogeological information isn’t available. However, the estimates will be prone to small scale errors (high variability resulting from large small scale variance, or in geostatistical terms, due to large nugget effects) - in the sense that only medium to large-scale properties are expected to be reproduced. A disadvantage of the VLF-R method is its shallow skin depth in the presence of high conductive overburdens. The data shown above indicates a penetration of 40 meters, which in many cases may not be enough to
reach the saturated zone. Higher penetration depth is possible if portable VLF sources are used. Unfortunately the amount of problems introduced with these sources is so high that their use isn’t possible in many cases (e.g., where horizontal electromagnetic wave propagation can’t be assumed). Higher skin depths are attainable for more resistive media. The average electrical resistivity of the limestone and dolomites in the study area is of 200 Ω m, which is in the lower end of the scale for the earth materials. The method is therefore of general application, but limited for particular cases.

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REFERENCES


