Exam-like questions: fuzzy sets

1. For the concepts below identify those that would benefit from fuzzy set based modeling; Characterize them.
   a) Real numbers close to 1;
   b) Real numbers very close to 1;
   c) Real numbers extremely close to 1;
   d) Odd integers lower than 5;

2. Characterize a set that represents the following notion
   a) Hot day; ELQ-fuzzy1.pdf
   b) Normal car engine temperature;
   c) Gravity acceleration;
   d) Tall Portuguese person;
   e) Tall Chinese woman.

3. Consider the following fuzzy sets both defined on $X = \{0, 1, 2, 3\}$ and the standard fuzzy set theory (i.e., intersession modelled by $\min$, union modelled by $\max$, and complement given by $n(x) = 1-x$).

   \[
   A = \{(0, 1), (1, 0.7)\} \\
   B = \{(0, 0.6), (1, 0.9), (3, 1)\}
   \]

   Compute and visualize the results of the following operations:
   i) Complement($A$), Complement($B$);
   ii) $A \cup B$, $B \cup A$
   iii) $A \cap B$, $A \cap \text{complement}(B)$
   iv) $A \times B$, $B \times A$

4. (Klir and Folger, 1988) Let $A$, $B$, and $C$ be fuzzy sets define on $X=[0,10]$, with membership functions:

   $\mu_A(x) = x/(x+2)$;
   $\mu_B(x) = 2^{-x}$;
   $\mu_C(x) = 1/(1+10(x-2)^2)$

   Compute and visualize the results of the following operations:
   i) $\bar{A}, \bar{B}, \bar{C}$
   ii) $A \cup B$, $A \cup C$, $A \cup C$
   iii) $A \cap B$, $A \cap C$, $A \cap C$
   iv) $A \cup B \cup C$
   v) $A \cap B \cap C$
   vi) $A \cap B$
5. Let $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ the fuzzy sets $A, B$ defined on $X$, s.t., $A = (0.2, 0.6, 0.7, 0.9, 1.0, 1.0)$ and $B = (0.0, 0.0, 1.0, 0.4, 0.6, 0.0)$

a) Compute the alpha-cuts for alpha equal to 0.2, 0.6, and 1.0;
b) Compute the scalar cardinality and the fuzzy cardinality of $A$ and $B$;
c) Classify $A$ and $B$ relatively to their convexity;
d) Verify that $A \cap \tilde{A}$ is different from the empty set and that $A \cup \tilde{A}$ is different from $X$ in standard fuzzy set theory.

6. If union is modelled by the triangular co-norm $s(x,y) = x + y - xy$, compute the dual De Morgan expression for modelling intersession.

7. Show that $n(x) = (1-x)/(1+ax)$, for $a>-1$ is involutive.

8. Which, if any, of the following functions can be used as fuzzy complete? Briefly justify
a) $f(x) = \cos(x)$;
b) $g(x) = \sin^2(x)$;
c) $h(x) = \cos(\pi/2 \times x)$;

9. Consider the axiomatic skeleton of triangular norms $t$ and co-norms $s$. Show that, for all $x, y$ in $[0,1]$,
a) $t(x, 1) = x$;
b) $t(x, 0) = 0$;
c) $s(x, 0) = x$;
d) $s(x, 1) = 1$;
e) $t (x, y) \leq \min (x, y)$;
f) $s(x, y) \geq \max(x, y)$;

10. Consider the axiomatic skeleton of triangular norms $t$ and co-norms $s$. Compute the expressions for
a) the lower triangular norm, $t_{\text{min}}(x,y)$;
b) the largest triangular co-norm, $s_{\text{max}}(x,y)$.