Software for slip-tendency analysis in 3D: A plug-in for Coulomb

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ABSTRACT

Slip-tendency analysis is a valuable tool in fault reactivation evaluation and seismic hazard assessment as it provides a means of quantifying the slip potential on mapped or suspected faults in a known or inferred stress field. We developed an interactive graphic tool to perform slip-tendency analysis. The application is written in MATLAB in the form of plug-ins for COULOMB, a graphic-rich deformation and stress change open-source software. In addition to identifying the faults most prone to reactivation, we compute and plot synthetic focal mechanisms from the direction and sense of likely slip. This allows compatibility between focal mechanisms and geological structures to be verified. Both individual faults and fault networks can be considered in three dimensions. The potential for slip depends on the prevailing stress field, the fault surface orientation and the coefficient of friction. These parameters are given interactively in a Windows environment. The application thus offers an easy-to-use graphical interface with the possibility of fast parameter modification and 3D visualization of the results.

1. Introduction

The reactivation of slip on pre-existing planes of weakness is of key interest to many geological phenomena, being implicit for instance in tectonic inversion theory and in the concept of stick-slip behavior. The analysis of reactivation is crucial in engineering geology where rock masses must be below the critical stress level needed to initiate sliding and in reservoir geology for selecting safe hydrocarbon traps. It also constitutes a valuable tool in seismic hazard assessment as it provides a means of quantifying the slip potential on mapped or suspected faults in a known or inferred stress field.

The foundations of the fault reactivation theory can be found in Jaeger (1969). He proposed a condition for reactivation based on the frictional resistance to sliding. It is usually assumed that after a shear fracture develops, the rock possesses no cohesion across the fracture plane; so the criterion for reactivation is the Navier–Coulomb for cohesionless faults, expressed as,

$$\tau = \mu (\sigma_n - p_f)$$

(1)

where \(\tau\) and \(\sigma_n\) are the shear and the normal stresses acting on the fault surface, respectively, \(\mu\) is the coefficient of the sliding friction and \(p_f\) is the pore fluid pressure. In general Eq. (1) applies only in the brittle part of the crust affected by frictional processes.

The stress conditions leading to fault slip have been analyzed in two key publications by Wallace (1951) and Bott (1959). The widely used Wallace–Bott hypothesis states that the direction of movement after a fracture is formed coincides with the direction of the maximum shearing stress within the fracture plane. For a given stress field, the direction of slip is determined by two factors: the orientation of the fault plane relative to the principal stress axes, and the stress difference ratio defined by the relative value of the three principal stresses.

The Wallace–Bott hypothesis is the basis of several inversion methods that estimate the stress field from the earthquake focal mechanisms or the fault–slip data (e.g. Angelier, 1979; Etchecopar et al., 1981; Gephart and Forsyth, 1984). Its validity may be questioned though, for it presupposes the homogeneity of the stress state and neglects factors such as fault interaction and fault block rotation. Nevertheless, it has been shown that it can be used as a first-order approximation since the deviation between the predicted and observed slip directions is on average less than 10° (Zoback and Zoback, 1980; Rebai et al., 1992; Dupin et al., 1993; Pollard et al., 1993; Pascal, 2002).

Direct methods follow the reverse procedure to inversion methods since they assume an a priori knowledge of the applied stress tensor. Knowing the fault geometry these methods are often used to predict the slip directions along faults as induced by stresses. An open-source software implementation for the calculation and graphical presentation of fault slip is already available to the community (Pascal, 2004). A complementary approach is to evaluate the slip potential of faults. Several graphical and analytical techniques serve this purpose, growing in complexity from the Mohr circle to triangular diagrams analogous to those used in petrology (Twiss and Moores, 1992; Yin and Ranalli, 1992). One of the most intuitive techniques is the slip-tendency analysis introduced by Morris et al. (1996), which
we follow in this paper and describe in detail in Section 2. This technique has been proven to be a robust method of predicting fault instability and preferred orientations of reactivation in several tectonic settings (e.g. Lisle and Srivastava, 2004).

The original representation of the slip tendency presented by Morris et al. (1996) consisted of an equal-angle lower hemisphere projection of poles of all possible fault orientations, contoured by magnitude of normalized slip tendency. Alainz-Alvarez et al. (1998) proposed a similar graphical technique using as reference the rupture strength of intact rock. A more insightful graphical representation of slip tendency would be a 3D representation of the fault surfaces showing the calculated slip tendencies in a color coded scale. This is the purpose of the software we described here.

In the present paper we make use of the framework of COULOMB to compute and represent the slip tendency in 3D. COULOMB is a popular deformation and stress change software used in earthquake, tectonic, and volcanic research and teaching (Lin and Stein, 2004; Toda et al., 2005). It allows the exploration of the stress transfer among mapped faults, and examining how an earthquake promotes or inhibits failure on nearby faults. The calculations are made in an elastic half-space with uniform elastic properties following Okada (1992). The great advantage of this integrated approach is the possibility of coupling the slip-tendency analysis to the COULOMB calculations of static displacements, strains and stresses caused by fault slip. Moreover, the implementation of the slip-tendency analysis in this framework takes advantage of the ease of input, rapid interactive modification and intuitive visualization of the results that characterizes COULOMB.

2. The slip-tendency concept

In terms of the effective stress \( \sigma = \sigma_0 - p_0 \), which incorporates the effect of the pore fluid pressure (see Eq. (1)), the critical condition for sliding on a pre-existing plane of weakness can be written as

\[
\mu = \frac{\tau}{\sigma}
\]

The slip tendency of a surface is defined as the ratio of the shear stress to the normal stress on that surface (Morris et al., 1996).

\[
T_s = \frac{\tau}{\sigma}
\]

It is therefore clear that the slip tendency equals the coefficient of sliding friction. The fault planes that will more likely slip are those with a high ratio of shear to the normal effective stress, close in value to \( \mu \). The slip-tendency analysis is based on the fact that the slope of the failure criterion, i.e. the coefficient of friction, may span a range of values limited by the Byerlee’s experiments. Generally \( \mu \) is in the range 0.6–0.85 (Byerlee, 1978). Fixing the stress difference ratio (Mohr’s circle diameter), we find a variety of combinations \((\theta, \mu)\) which make slip viable (Fig. 1). In a region dominated by a particular rock type, the assumption of a specific \( \mu \) determines the optimum angle \( \theta \) for sliding (Jaeger, 1969), \( 2 \theta = \tan^{-1}(1/\mu) \). This is the most favourable orientation of the fracture plane relative to the direction of maximum compression. In this plane the slip tendency is maximum, i.e. \( T_s = T_s^{\text{max}} \). A normalized slip tendency varying between 0 and 1 is defined by dividing the slip tendency by its maximum possible value \( T_s = T_s/T_s^{\text{max}} \). The normalized slip tendency then ranges from 100% near the ideal fault orientation to 0% in the principal stress directions.

If we know the stress tensor completely (principal stress directions and principal stress values) the shear and the normal stresses, \( \tau \) and \( \sigma \), acting on a given plane are simply (Jaeger, 1969)

\[
\tau^2 = (\sigma_1 - \sigma_2)^2 P m^2 + (\sigma_2 - \sigma_3)^2 m^2 n^2 + (\sigma_3 - \sigma_1)^2 n^2 l^2
\]

\[
\sigma = P \sigma_1 + m^2 \sigma_2 + n^2 \sigma_3
\]

where \( \sigma_i \) \((i=1,2,3)\) are the principal stress values and \((l,m,n)\) are the direction cosines of the normal to the plane in the principal stress system. Most commonly, however, we only know the principal stress directions and the principal stress difference ratio, \( \sigma_1 - \sigma_2 \), acting on a given plane are simply (Jaeger, 1969)

\[
\phi = 1 - \frac{R}{\sigma_1 - \sigma_2}
\]

we can write the principal stress values in terms of \( \phi \) and of two unknown parameters \( k_1 \) and \( k_2 \), related to the size and location of the 3D Mohr circles (Fig. 1),

\[
\sigma_1 = k_1 + k_2
\]

\[
\sigma_2 = k_1 \phi + k_2
\]

\[
\sigma_3 = k_2
\]

Upon substitution of these expressions in the criterion for frictional sliding (Eq. (2)), and assuming that the frictional sliding envelope is tangential to the \( \sigma_1 - \sigma_2 \) Mohr circle (i.e. using the

Fig. 1. Sketch of normalized slip tendency in relation to the Mohr diagram. The set of planes oriented at \( \pm \theta \) relative to maximum compression will slip before a new fracture making an angle \( \pm \theta \) with respect to maximum compression will form. Principal stresses \( \sigma_1, \sigma_2, \sigma_3 \) can be expressed in terms of unknown parameters \( k_1 \) and \( k_2 \).
identity $\tan \varphi = 1/\tan 2\theta$, it can be shown that

$$k_2 = k_1 \varphi - \sigma_1$$

and the parameter $k_2$ can be eliminated from the principal stress value equations. This allows rewriting the shear and the normal stresses acting on a given plane as

$$\tau = [(1 - \phi)^2 \mu^2 + \phi^2 m^2 n^2 + n^2 \mu^2]$$

$$\sigma = k_1 \left( \frac{\phi^2 + 1}{2} - (1 - \phi) m^2 - n^2 \right)$$

It follows that the slip tendency $T_s$ computed from these expressions is independent of the choice of the unknown parameter $k_1$. Thus, the relative magnitude of the shear and the normal stresses (slip tendency) is independent of the absolute magnitude of the principal stresses and only depends on the orientation of the fracture plane in the stress field and on the principal stress difference ratio. In this way we downgrade the importance of knowing the stress difference $\sigma_1 - \sigma_3$ and the pore fluid pressure, which are usually poorly constrained.

The likely direction and sense of motion can be estimated from the direction of the maximum resolved shear stress on the fracture plane. In the principal stress system the components of the shear stress $\tau$ vector can be written in terms of $\Gamma = m^2 + (1 - \phi)n^2$ as

$$\tau_1 = -\Gamma l$$

$$\tau_2 = -(\Gamma - 1 + \phi) m$$

$$\tau_3 = -\Gamma n$$

The normalized shear stress vector is also independent of the choice of $k_1$. Knowing the strike and dip of the fault surface and the direction of slip on the fault plane, one can readily compute slip-tendency analysis.

3. Input and output variables

The slip-tendency theory predicts that the orientation of the fault planes most prone to reactivation is mainly determined by the orientation of the stress tensor, the stress shape ratio $\phi$ (or the principal stress difference ratio $R$), the friction coefficient $\mu$ and the pore fluid pressure $P_f$ (which is implicitly taken into account by the use of the effective stress). The influence of these parameters on the slip tendency can be found in several key publications that use slip-tendency contours plotted on equal area or equal-angle stereonets. These stereoplots clearly illustrate how the patterns and the values of the slip tendency can vary with the stress shape ratio (Morris et al., 1996; Lisle and Srivastava, 2004; Morris and Ferril, 2008), the pore fluid pressure (Collettini and Trippetta, 2007; De Paola et al., 2007) and the friction coefficient (e.g. Lisle and Srivastava, 2004).

In the software, the input parameters required to perform a slip-tendency analysis are presented as follows: (1) the principal stress directions; (2) the stress difference ratio $R$; (3) the geometry of the fault system and (4) the coefficient of friction.

The principal stress directions $X_1, X_2, X_3$ are referenced to the right-handed coordinate system East, North and Up (Fig. 2a). The direction of each principal axis is defined in terms of strike and dip (Fig. 2a). The strike $\Psi$ is measured in the horizontal plane in degrees clockwise from North and is in the range $0 \leq \Psi \leq 360^\circ$ or $-180^\circ \leq \Psi \leq 180^\circ$. The dip $\delta$ is measured in the vertical plane in degrees down from the strike direction and is in the range $0 \leq \delta \leq 90^\circ$.

The principal stress magnitudes are denoted by $\sigma_1, \sigma_2, \sigma_3$ and by definition $\sigma_1 \geq \sigma_2 \geq \sigma_3$. We follow the sign convention used in geology, i.e. compression is positive, so the principal stresses are ordered from the most to the least compressive. Although the value of principal stress magnitudes may be specified, only the value of the stress difference ratio $R$ (see Eq. (6)) is relevant to the slip-tendency analysis.

The stress field is not required to be Andersonian (that is, the principal stresses may deviate from the vertical and the horizontal) but it is assumed to be homogeneous. The homogeneous assumption implies that the “regional stress field” at a given scale needs to be consistent with the scale of the fault models.

The faults are assumed to be planar and do not interact. They are specified according to Aki and Richards (1980) also in terms of strike and dip (Fig. 2b). The fault geometry can either be imported or be drawn interactively.

The program can analyse tens of faults in very short time and returns graphical results. Two graphic windows can be outputted: (1) a 3D view of the fault planes showing the calculated normalized slip tendency and (2) a separate window (which can be exported as an image) displaying the computed synthetic focal mechanism for a selected fault. The program is also able to store the numerical parameters of the synthetic focal mechanisms in an ASCII file.

4. Package description

The slip-tendency analysis is performed under the Coulomb program, which is a Matlab application. The program runs under Matlab 7.x or higher and is freely available from http://www.coulombstress.org. Once you have the program you will find the plug-ins directory under the Coulomb 3.x folder. The slip-tendency plug-ins we developed must be placed in this directory in order to be used under Coulomb. Each plug-in for Coulomb constitutes a separate Matlab (.m) file. We developed two primary functions given as follows:

(1) SLIP_TENDENCY: Computes the slip tendency for all faults and plots a 3D view of the fault planes showing the slip tendency in a color coded scale.
(2) SYNTHETIC_SLIP: Computes the slip tendency for all faults and displays those with slip tendency greater than 0.5 as a dashed line in a 2D grid drawing. It allows selection of any fault and choosing to display the synthetic focal mechanism in a separate window.

The second group of functions is subsidiary to the primary plug-ins. They are as follows:

a) PRE_SLIP_TENDENCY: Checks the conditions before computing the slip tendency. It tests if the principal stress axes are orthogonal and if the regional stresses are in the correct order. It also allows the stress difference ratio to be checked and modified if necessary.

b) EVALUATE_SLIP_TENDENCY: Given the principal stress directions and the stress difference ratio, this function computes the normalized slip tendency for each fault plane. The slip tendency is maximum for planes making an optimum angle with the direction of \( \sigma_1 \).

c) FOCAL_MECHANISM: For any selected fault this function computes the direction and the sense of displacement predicted on that particular fault plane. The predicted displacement is expressed in terms of a synthetic focal mechanism defined by a particular strike, dip and rake.

d) PATCH_FOCAL_MECHANISM: Computes the geometric parameters of the focal mechanism beach balls on an equal area projection. This function has originally been developed for program Mirone (Luis, 2007).

e) DRAW_FOCAL_MECHANISM: Displays the synthetic focal mechanism for the selected fault in a separate window. The synthetic focal mechanisms can be exported to a PNG image and/or to an ASCII file using the GMT (Wessel and Smith, 1991) Aki and Richards format. In the last case, it is possible to append several selected focal mechanisms, corresponding to several selected faults, into a single file. Note that the output coordinates for the focal mechanisms are in the units displayed in the Coulomb window.

5. Instructions for use

Launch Matlab on your computer. In the Matlab menu bar top line change the path to the Coulomb 3.x folder and next write Coulomb in the Matlab command window. This will display the pop-up Coulomb 3.x window. To open an existing Coulomb input file choose Input -> Open existing file. In the pop-up open input file window change to subdirectory input files and choose one of the examples, then hit \( \text{return} \). The program will automatically plot a 2D grid view of the fault network. At this stage, the coordinates of the model should be converted if needed using Functions -> Tools -> Convert lat/lon to Cartesian.

The parameters in the input file, which are relevant to the slip-tendency analysis, in addition to the fault’s geometry, are the regional stress field parameters and the regional coefficient of friction. You can change these by choosing Functions -> change parameters -> all input parameters. To execute the plug-ins, you have to return to the Matlab Command Window and write SLIP_TENDENCY or SYNTHETIC_SLIP. In either case a pop-up menu appears immediately after, asking to confirm the stress difference ratio.

When the plug-in SLIP_TENDENCY is executed a 3D view of the fault planes appears showing the slip tendency for each fault along with the slip-tendency color scale. When the plug-in SYNTHETIC_SLIP is executed a 2D view of the fault network...
appears showing the faults with slip tendency greater than 0.5 as dark green dashed lines. A right click on a fault allows selecting the option change parameters or focal mechanism (Fig. 3). The change parameter option is a Coulomb built-in function that allows modifying the fault parameters. Choosing this option will exit the plug-in. The focal mechanism option opens a new window showing the synthetic focal mechanism predicted for the selected fault under the given stress field.

Fig. 5. (a) Regional stress field estimated by Stich et al. (2006). Arrows represent horizontal projections of the maximum (red) and minimum (yellow) principal stress directions. Bathymetry and fault traces as in Fig. 3. and (b) Horseshoe region grid in 3D view with faults colored by their slip-tendency value. Plotted color bar is scaled according to calculated maximum of slip tendency. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
6. Example of application

As a case study we apply the method to the Horseshoe region in the Gulf of Cadiz (Fig. 4). The Gulf of Cadiz is characterized by distributed seismicity associated with the Africa–Eurasia plate convergence. The largest events concentrate in the Horseshoe Abyssal Plain (e.g. 29.2.1969, Ms=8 and 12.2.2007, Mw=6.1) and occur at ~30–40 km depth (e.g. Bufforn et al., 2004; Stich et al., 2007). The earthquakes have focal mechanism solutions consistent with predominant NNE–SSW striking thrusts and WNW–ESE trending dextral strike-slip faults, recognised after swath-bathymetry, sidescan sonar, seismic reflection and refraction surveys in the area (e.g. Gracia et al., 2003; Sartori et al., 1994; Zitellini et al., 2001, 2004; Terrinha et al., 2003;

![Image](image-url)

**Fig. 6.** (a) Possible regional stress directions proposed in this study AND (b) tendency for fault reactivation (slip-tendency value) is greater for the Horseshoe (HsF) and St. Vicent (SVF) faults in agreement with seismicity observations (see Fig. 4).
However, the relation between the earthquakes and the faults remains unclear because the faults are only imaged down to ~6 km beneath the seafloor in the seismic reflection profiles (ibid.), whereas most earthquakes have focal depths between 10 and 30 km. It is therefore interesting to explore if the earthquake ruptures in this region nucleate on planes successfully predicted by the slip-tendency theory. The 3D geometric model of the fault pattern has been created from the fault trace lines (Fig. 4) identified by Terrinha on the grounds of seismic reflection profiles and high resolution swath-bathymetry data (unpublished results, 2007). It constitutes a first-order approximation of the real structure since we used 2D planar fault geometries with dipping angles predicted by theory, i.e. vertical strike-slip faults, thrusts dipping at ~30° and normal faults dipping at ~60°. These fault angles are supported by the seismic reflection profile interpretations. The main faults can be divided into two groups: thrusts with NNE–SSW orientation (MPF, SVF, HsF) and strike-slip faults with WNW–ESE orientation (L1, L2, L3). The Pereira de Sousa fault (PSF) is the exception and the only SVF, HsF) and strike-slip faults with WNW–ESE orientation (L1, L2, L3). The Pereira de Sousa fault (PSF) is the exception and the only normal fault considered in this study. The faults extend to a maximum depth of approximately 30 km, which is the estimated depth of the brittle–ductile transition in this region (Neves and Neves, 2009).

The regional stress tensor in the area between the Gorringe Bank and the Straits of Gibraltar has recently been estimated by Stich et al. (2006) based on 11 published seismic moment tensors. They determined that the tectonic regime is transpressive with NW–SE shortening and the strike and dip of the three principal stress axes are for \( \sigma_1 (\Psi = 310^\circ, \delta = 22^\circ) \), for \( \sigma_2 (\Psi = 212^\circ, \delta = 19^\circ) \) and for \( \sigma_3 (\Psi = 85^\circ, \delta = 61^\circ) \). The horizontal projection of \( \sigma_1 \) and \( \sigma_3 \) is shown in Fig. 5a. The stress difference ratio \( R \) is estimated to be 0.5 (Stich et al., 2006).

The application of the slip-tendency analysis (SLIP_TENDENCY plug-in) to the fault network model using the Stich et al. (2006) stress tensor gives the results shown in Fig. 5b. In this case the highest probability of slip occurs for the Pereira de Sousa fault (PSF), which is a fault without a significant seismic record (Fig. 4). However, this is an intriguing fault because the structural interpretation based on seismic profiles and sonar images suggests that the PSF is either being reactivated as a normal fault or is suffering passive uplift of the footwall (Terrinha et al., 2003). Therefore, it is possible that the PSF is in an interseismic period.

Small changes in the orientation of the stress tensor have a strong impact on the slip-tendency distribution. Taking into account that in the Horseshoe region several earthquake source estimates reveal that the direction of maximum compression is nearly N–S (Fukao, 1973; Bufforn et al., 1988; Stich et al., 2007) and clearly different from the regional plate convergence direction (~WNW–ESE), we consider another stress tensor where the direction of maximum compression has been slightly rotated (Fig. 6a). Under these conditions the Pereira de Sousa fault has a slip tendency less than 0.5 and the highest probabilities of slip occur for the St. Vicent fault (SVF) and Horseshoe fault (HsF) (Fig. 6b). This is more in agreement with the seismicity observations and suggests that the direction of maximum compression in the Horseshoe region may be N340W instead of the regional estimate of N310W made by Stich et al. (2006). We also tested the sensitivity of the results to changes in the stress shape ratio \( R \) (varying between 0 and 1). Similar results were obtained irrespective of the \( R \) value, showing that in this case the orientation of the stress tensor is a dominant factor. This example demonstrates how the slip-tendency analysis can be used as an additional component of stress-inversion schemes to estimate the local lateral changes of the stress tensor.

The synthetic focal mechanisms provide useful insights about the style of fault reactivation. For the stress tensor shown in
7. Concluding remarks

The interest of coupling the slip-tendency analysis with COULOMB, a widely used freeware MATLAB program designed to compute and display static displacements, strains and stresses caused by fault slip (Lin and Stein, 2004; Toda et al., 2005), led us to develop a plug-in for this program. The program COULOMB is particularly effective to explore how changes in Coulomb stress associated with one or more earthquakes may trigger subsequent events (e.g. King et al., 1994). Performing a slip-tendency analysis for a network of faults previously to COULOMB computations greatly helps to identify source faults for potential main shocks.

A tool to compute and plot the focal mechanism solution associated with the predicted slip direction for a given fault is also presented. The representation of synthetic focal mechanisms provides an easy way of verifying the compatibility between earthquakes with observed focal mechanisms and mapped faults.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found in the online version at doi:10.1016/j.cageo.2009.03.008.

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