Nonlinear Dynamics of a Liénard Delayed-Feedback Optoelectronic Oscillator

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Outline

- Architecture of an Optoelectronic Oscillator (OEO).
- Resonant Tunneling Diode (RTD) - a high frequency negative resistance oscillator.
- RTD Optoelectronic Oscillator (RTD-OEO).
- Nonlinear dynamic regimes:
  - stable free-running oscillations;
  - synchronization and chaos.
- Nonlinear dynamical model:
  - Non-autonomous Liénard oscillator with additive white Gaussian input noise;
  - Laser diode single mode rate equations;
  - Time delayed feedback system using delay differential equations (DDEs);
- RTD-OEO applications in microwave-photonics.
- Conclusion.
Optoelectronic Oscillator (OEO)

Typical OEO

Resonant tunneling diode based OEO (RTD-OEO)

The proposed optoelectronic oscillator (OEO) comprises:
- a resonant tunneling diode photo-detector (RTD-PD) for O/E conversion;
- a laser diode (LD) for E/O conversion;
- an optical fiber delay line for phase-noise reduction.

Main advantages:
- No need of RF and optical amplifiers;
- Low power requirements.
Negative Resistance Elements

**Diagram:**
- **Conventional ways of getting negative conductance**
- **Dynamic resistance** $R>0$ → **Power** $P_{\text{diss}}>0 = \text{losses}$
- **Dynamic resistance** $R<0$ → **Power** $P_{\text{diss}}<0 = \text{GAIN}$

**Equations:**
- $I = GV$
- $R = 1/G < 0$
- slope $< 0$
- $\Delta V > 0 \rightarrow \Delta I < 0$: $G < 0$

**Notes:**
- Two bipolar transistors and five positive linear resistors.
Resonant Tunneling Diode (RTD)

- Double Barrier Quantum Well (DBQW) Resonant Tunneling Diodes (RTDs) are nonlinear devices that use quantum effects to produce Negative Differential Resistance (NDR).

Typical DBQW-RTD Epilayer Structure

Current-Voltage (I-V) curve

I

V

Slope < 0 (NDR)

P_{\text{diss}} < 0 (GAIN)
How does an RTD works?

Conduction band profile under applied voltage

Zero Bias (i)  
Resonance (ii)  
Off Resonance (iii)

RTD I-V characteristic

VCO operating around 2 GHz

RTD:
- the NDR provides wide bandwidth electronic gain;
- self-sustained oscillations up to THz frequencies;
- voltage controlled oscillator (VCO).

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Hybrid RTD-OEO Prototype

RTD optical waveguide photodetector

RF in
RF out
(to the PCB strip line)

Light in

Light out

Au wire

0.5 mm

Delay Line

Bias-T

Electrical Input

Optical Input

RTD-PD

LD

Optical Output

Electrical Output

Current (A)

RTD-OW

LD

RTD-OW-LD

NDR

NDR

Voltage (V)

0.0

0.07

0.08

0.06

0.05

0.04

0.03

0.02

0.01

0.00
Liénard Oscillator Model (ODEs)

- Nonlinear dynamics model of the RTD-PD oscillator perturbed by electrical and optical-modulated signals using ordinary differential equations (ODEs).

Non-autonomous Liénard oscillator model using ODEs:

\[
\begin{align*}
\frac{dV(t)}{dt} &= \frac{1}{C} \left[ I(t) - F(V) - I_{ph} \right] \\
\frac{dI(t)}{dt} &= \frac{1}{L} \left[ V_{DC} + V_{AC} \sin(2\pi f_{in}t) - RI(t) - V(t) \right]
\end{align*}
\]
RTD-OEO Model (DDEs)

- The proposed optoelectronic oscillator (OEO) model comprises:
  - Liénard oscillator with additive white Gaussian input noise;
  - laser diode (LD) single mode rate equations;
  - time delayed feedback system using delay differential equations (DDEs).

RTD-OEO dimensionless DDEs:

\[
\begin{align*}
\frac{dx(t)}{dt} &= \frac{1}{\mu} \left[ y(t) - f(x) - \chi \xi_y(t) - \eta s(t - \tau) \right] \\
\frac{dy(t)}{dt} &= \mu \left[ \nu_0 + \nu \sin(z(t)) - \gamma y(t) - x(t) \right] \\
\frac{dz(t)}{dt} &= 2\pi f_{in} \\
\frac{n(t)}{dt} &= \frac{1}{\tau_n} \left\{ \frac{i}{i_{th}} - n(t) - \frac{n(t) - \delta}{1 - \delta} \left[ 1 - \varepsilon s(t) \right] s(t) \right\} \\
\frac{s(t)}{dt} &= \frac{1}{\tau_s} \left\{ n(t) - \delta \left[ 1 - \varepsilon s(t) \right] s(t) - s(t) + \beta n(t) \right\}
\end{align*}
\]

Block Diagram (variables and control parameters)
Experiment: stable free-running oscillations

- Free-running oscillations and self-injection locking. The average in-fiber optical power was ~5 dBm.

The mode spacing of the free spectral range (FSR) is inversely proportional to the fiber length:

$$FSR = \frac{c}{n_f L} = 167 \text{ kHz}$$

- $n_f$ – fiber refractive index
- $c$ – velocity of light
- $L$ – optical fiber length

1 MHz Span, 10 kHz RBW.
Model: self-injection as a function of time delay

- The model predicts the FSR (side modes) associated with the time delay line and spurs level (single mode suppression ratio - SMSR).

Delay, $\tau=6.49 \, \mu s$ ($L=1.219 \, km$)
- Delayed feedback strength, $\eta = 5 \times 10^{-4}$
- Amplitude of noise, $\chi = 7 \times 10^{-5}$

Delay, $\tau=8.33 \, \mu s$ ($L=1.624 \, km$)
- Delayed feedback strength, $\eta = 5 \times 10^{-4}$
- Amplitude of noise, $\chi = 7 \times 10^{-5}$
Experiment: Synchronization

Limit cycle

Laser Diode ↔ RTD
DC Port ↔ RF in → RF out
Optical out

Frequency division by 1

Frequency division by 2

Limit cycle

RF 0.87 GHz ↔ Electrical
Optical

RF 1.9 GHz ↔ Electrical
Optical

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Arnold’s Tongues synchronization diagram

Arnold’s Tongues Diagram
($p = f_{in}/f_0$ in the very weak injection condition)

Adler’s equation – the short version of Arnold’s tongues

\[ \Delta f = \frac{f_0}{2Q} \sqrt{\frac{P_{inj}}{P_0}} \]

$\Delta f$ – locking range
$f_0$ – oscillator frequency
$P_{inj}$ – power of injected signal
$P_0$ – output power of oscillator
$Q$ – oscillator quality factor

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Experiment: Route to chaos

Quasi-periodic signals

Chaos

Optical out

Laser Diode

RTD

DC Port

RF in

RF out

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Model: synchronization and route to chaos

- As the external frequency is varied a stable period-$n$, $n=1, 2,\ldots$ is obtained, followed by a quasi-periodic/chaotic region.

Synchronization

Chaos

Frequency ratio, $f_{in}/f_0$

(bifurcation map)
Model: Lyapunov exponents

- We used the Lyapunov Characteristic Exponents (LCEs) to analyze the stability of the Liénard dynamical system.

\[ \nu = 0.15 \]

\[ \nu = 0.25 \]

chaotic regions (exponent > 0)
RTD-OEO applications in RF-photonic systems

- Wide range of microwave-photonic applications:
  - VCO with electrical and optical outputs for photonic RF systems;
  - RF carrier amplification and distribution in photonic links;
  - Clock and carrier recovery of incoming random data;
  - Chaotic generator for optical chaos communication systems.
Demonstration of a novel optoelectronic oscillator (OEO) comprising a negative resistance RTD oscillator, a laser diode and optical fiber delay line.

The RTD-OEO experimental dynamical regimes include very stable free-running oscillations, synchronization to external signals and chaotic output.

We have presented a time delayed-feedback Liénard dynamical system that is capable of predicting the RTD-OEO experimental dynamical regimes.

The Liénard’s model can be used to describe a wide class of oscillators derived from negative resistance oscillators with time-delayed feedback, and to study their nonlinear dynamic characteristics.